

# Sensitivity of Cache Replacement Policies

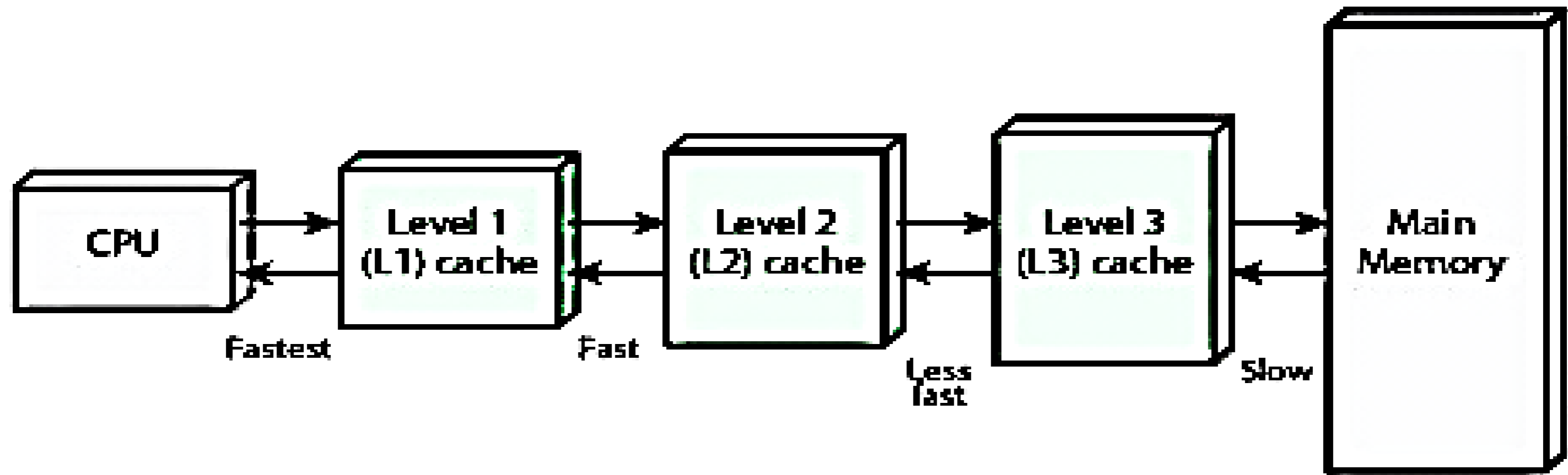
JAN REINEKE, DANIEL GRUND

Presented by Panchali Mukherjee  
Saarland University

Robustness of Hardware and Software Systems

# What is a cache?

- ❑ Memory unit.
- ❑ Conceals latency gap.
- ❑ Close to the CPU.
- ❑ Can access CPU faster.



(b) Three-level cache organization

Data found in cache: hit.

Data not found in cache: miss.

- ▶ Hit rate determines effectiveness.

# What does Cache Miss result in?

- ▶ Very high cache miss penalties.
- ▶ Cache performance influences overall performance.

# Worst Case Execution Time

The **maximum length of time** a task or set of tasks requires on a specific hardware platform.

# Worst-case Execution Time Analysis:

Large variance gets introduced into the execution time due to:

- Cache misses
- Pipeline stalls, etc

# Worst-case Execution Time Analysis:

WCET computation methods:

- Measurement-based timing Analysis
- Static Analysis.

# Static Analysis:

- Uses abstract model.
- Computes invariant.
- Gives upper bound on WCET.



# Static Analysis: Pros

- Gives good results for simple hardware.
- Efficient model.
- Accurate prediction.

# Static Analysis: Cons

- Complex hardware: error prone and laborious.
- Inaccurate model.
- Over-pessimistic result.

# Measurement Timing Analysis:

- Subset of real hardware states.
- Gives maximum of execution times measured.
- Underestimation of WCET.

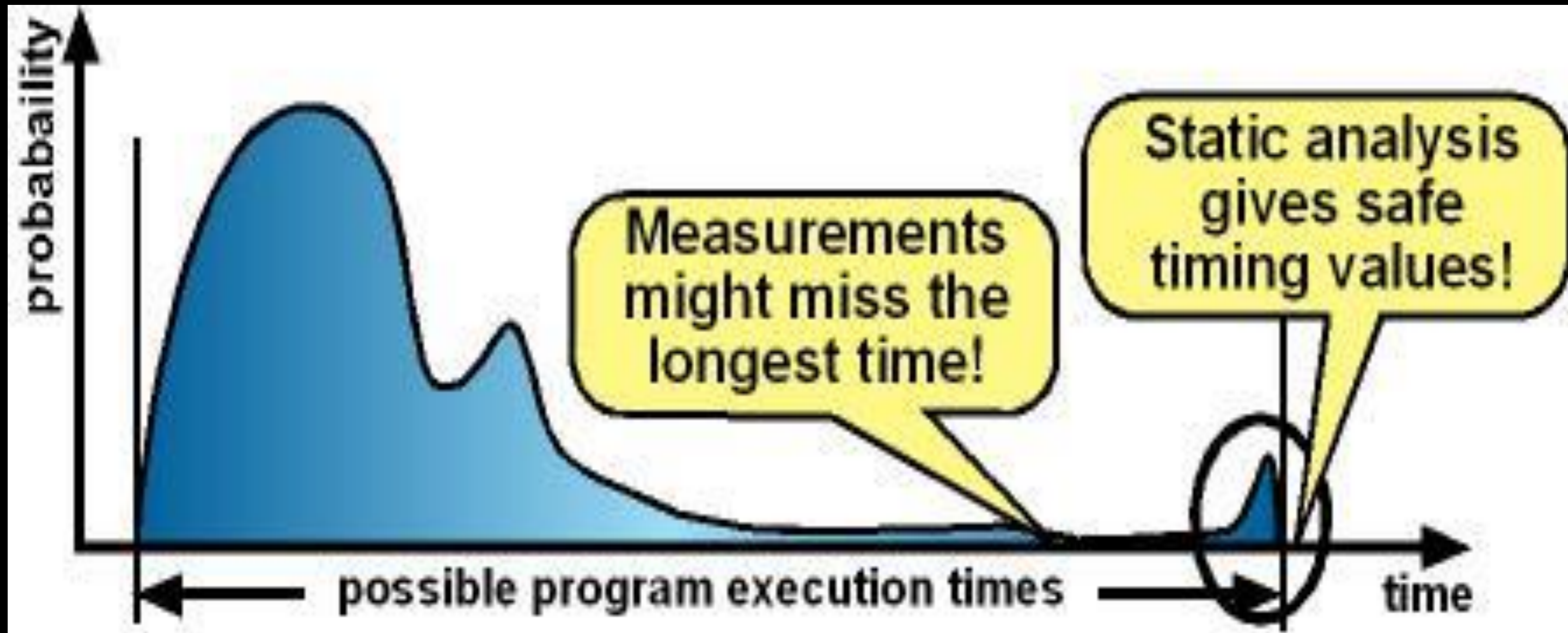
# Measurement Timing Analysis: Pro

- Gives good results for both simple and complex hardware.
- Precise estimate.
- Portable.

# Measurement Timing Analysis: Con

- May suffer from over-pessimism.
- Not sound.

# Worst-case Execution Time Analysis:



# Objective:

Influence of initial hardware state on cache effectiveness.

# Associativity:

The size of a cache set is called the associativity  $k$  of the cache.

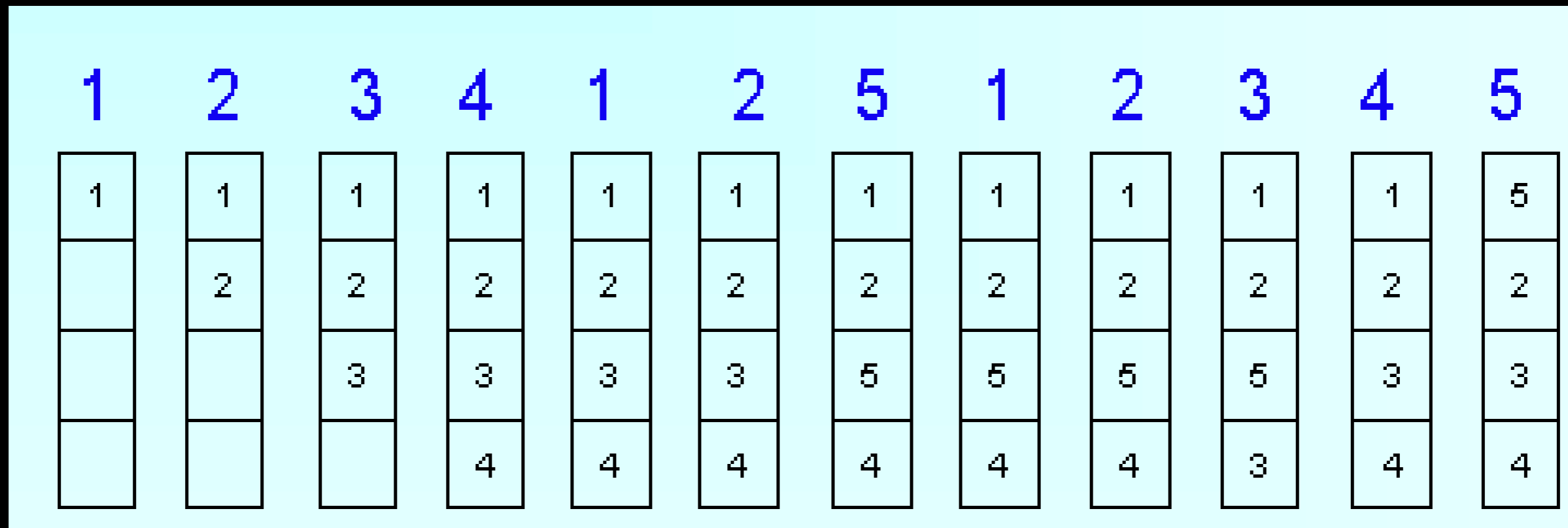


The index is used to find the set, and the tag helps find the block within the set.



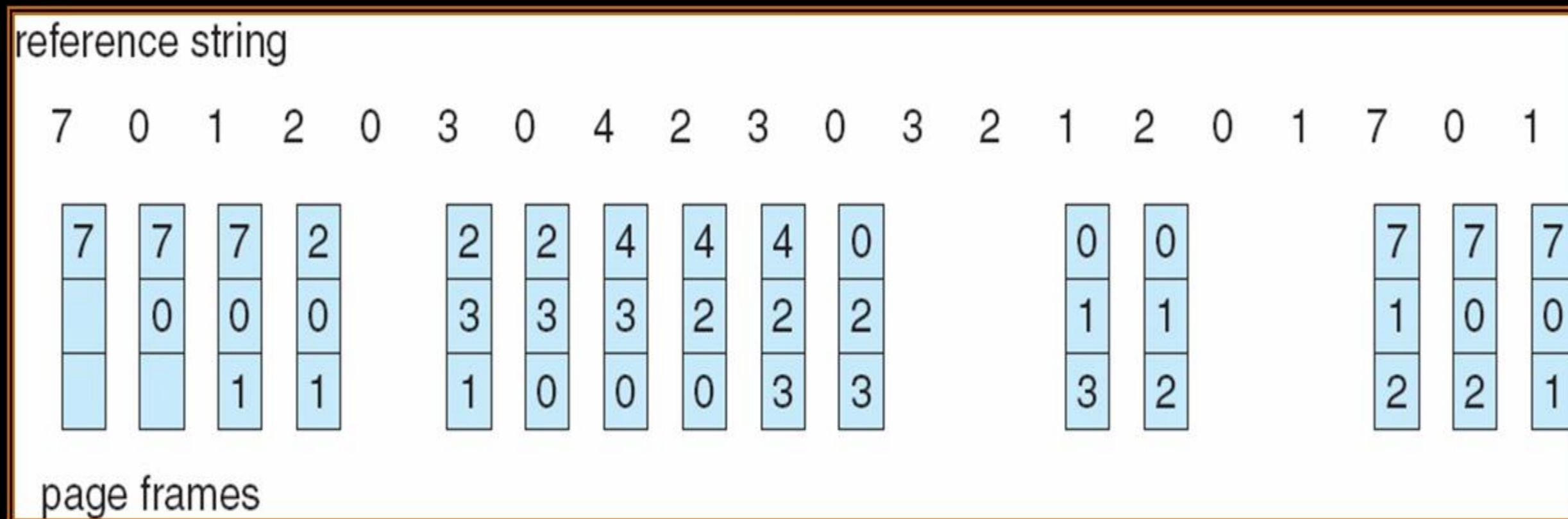
# Cache Replacement policies:

**LRU (least-recently-used):** The bit that has not been used for the longest period of time is replaced.



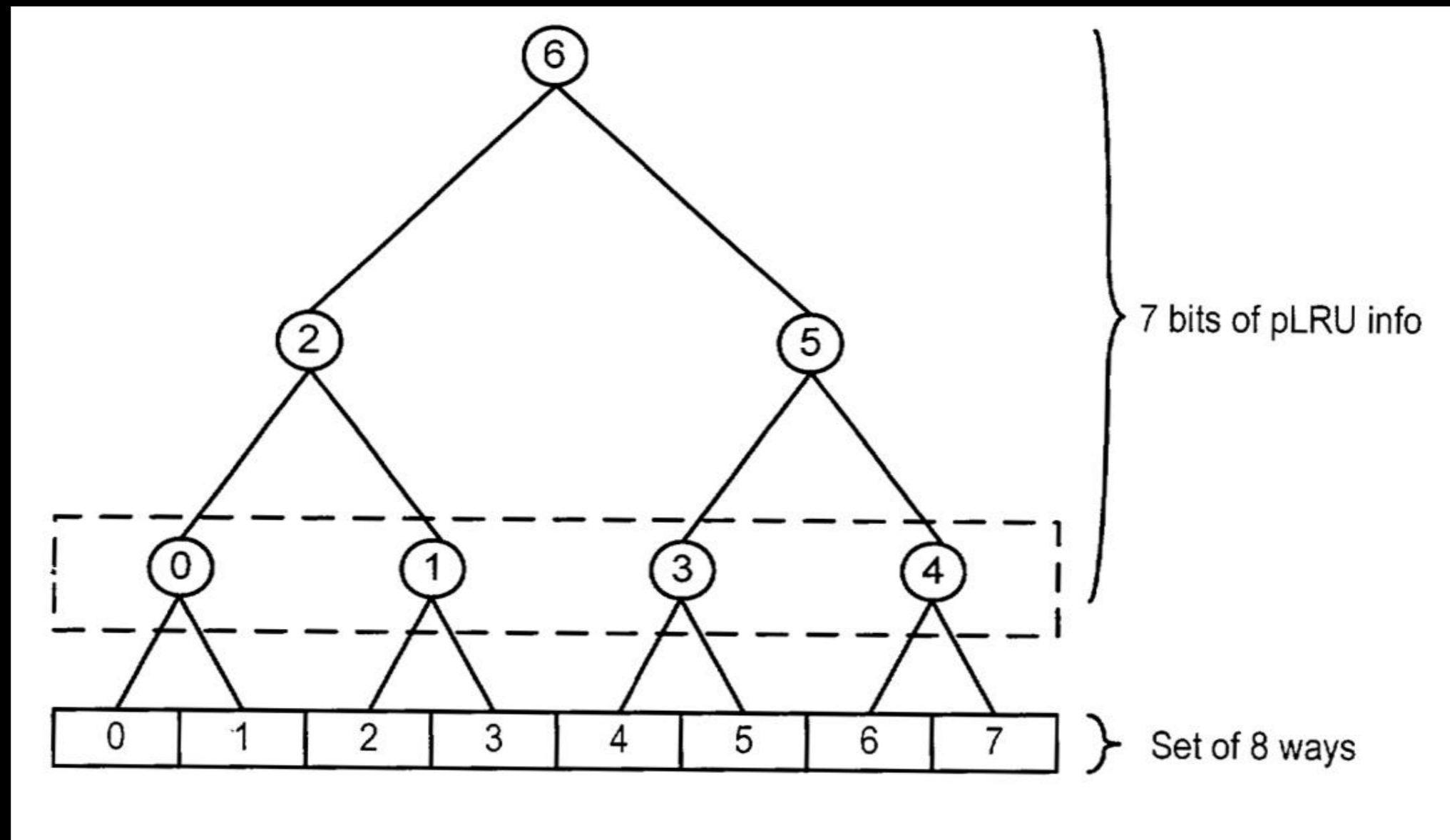
# Cache Replacement policies:

**FIFO (first-in-first-out):** The bit that was the first to enter the cache is replaced.



# Cache Replacement policies:

**PLRU (Pseudo-LRU):** Tree-based approximation of the LRU policy.



# Cache Replacement policies:

## MRU (most-recently-used):

- One recently-used bit per cache line.
- Cache line accessed—bit is set.
- Cache miss—first cache line without set bit selected,
  - block removed,
  - latest recently used bit set to 1.
  - all other bits reset to 0.

## Sensitivity:

$q, q'$  : cache states,

$s$ : access sequence,

$m_P(q, s)$ : number of misses,

$h_P(q, s)$ : number of hits,

$P$ : replacement policy used.

# Sensitivity:

**Miss-Sensitivity to State:** A policy  $P$  is  $k$ -miss-sensitive with additive constant  $c$ , if

$$m_P(q, s) \leq k \cdot m_P(q', s) + c$$

# Sensitivity:

**Hit-Sensitivity to State:** A policy  $P$  is  $k$ -hit-sensitive with subtractive constant  $c$ , if

$$h_P(q, s) \geq k \cdot h_P(q', s) - c$$

## Sensitivity:

**Sensitive Ratio:** The sensitive miss and hit ratios  $s_p^m$  and  $s_p^h$  of  $P$  are defined as:

$s_p^m = \inf \{k \mid P \text{ is } k\text{-miss-sensitive}\}$  and

$s_p^h = \sup \{k \mid P \text{ is } k\text{-hit-sensitive}\}.$



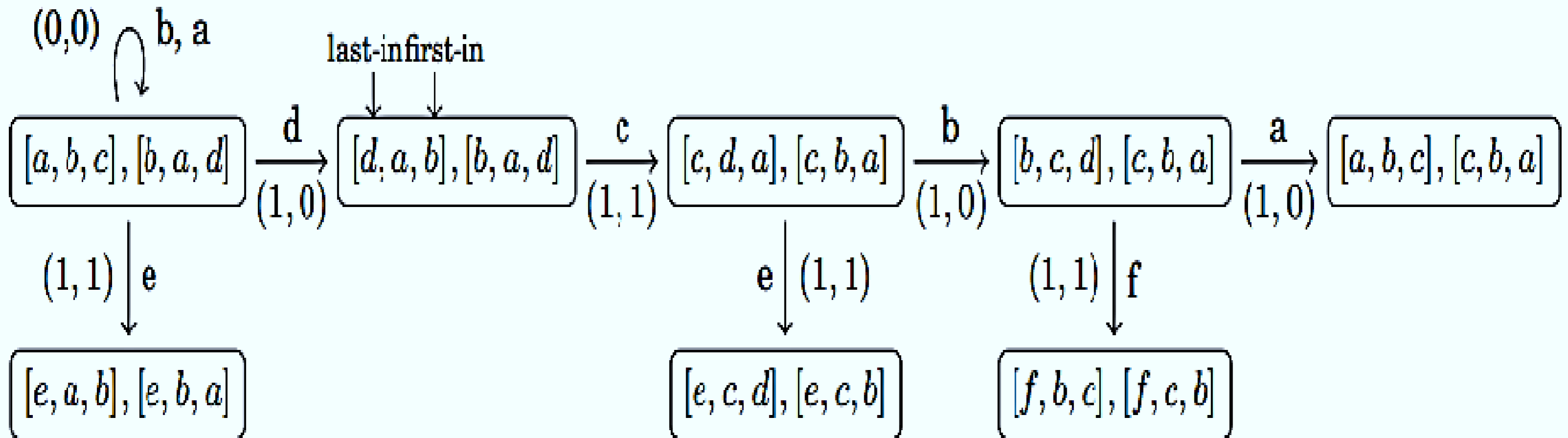
# Compute Sensitive Ratios:

**RELACS:** Automatically computes sensitive ratios.

## Compute Sensitive Ratios:

**Transition system:** A system whose states are pairs of cache states, and whose transitions reflect the effect of a memory access on both of the cache states.

# Compute Sensitive Ratios:



Sensitive ratios depend on the number of misses (hits) on paths through the transition system.

# Induced Transition System:

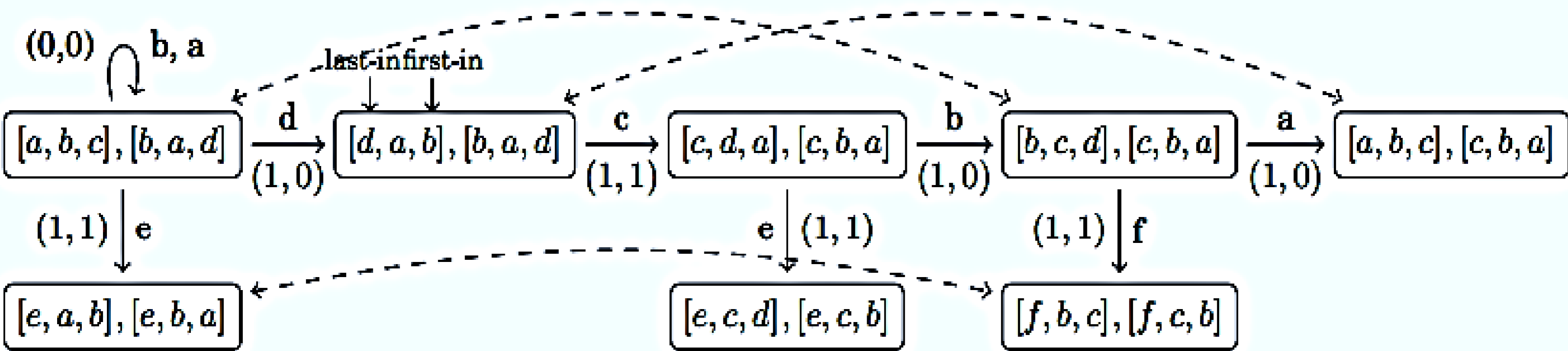
- ▶ **Induced Transition System:** A policy  $P$  induces a labeled transition system  $T_P = (S_P, R_P)$ , where:
- ▶  $S_P = \{(q, q') \mid q \in C_P, q' \in C_P\}$ , are the states, which are pairs of cache states of policy  $P$ ,
- ▶  $R_P = \{((p, q), (m_p, m_q), (p', q')) \mid (p, q) \in S_P, a \in B,$
- ▶  $(p', q') = \text{update}_{P,P}((p, q), \langle a \rangle)$
- ▶  $(m_p, m_q) = m_{P,P}((p, q), \langle a \rangle)\}$

# Induced Transition System:

If set of memory blocks infinite, the induced transition system is infinitely large.

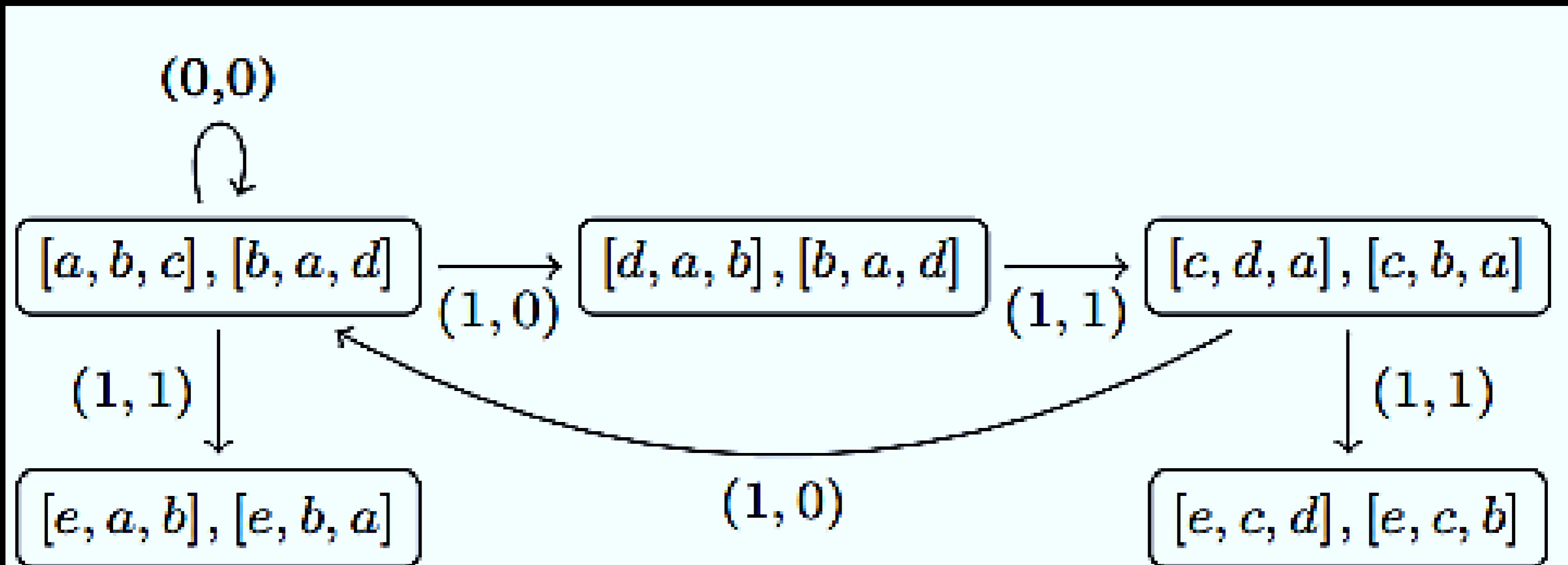
Solution: Finite Quotient Structure.

# Merging Equivalent States:



(a) Dashed lines connect equivalent states according to the equivalence relation  $\approx$ .

# Merging Equivalent States:



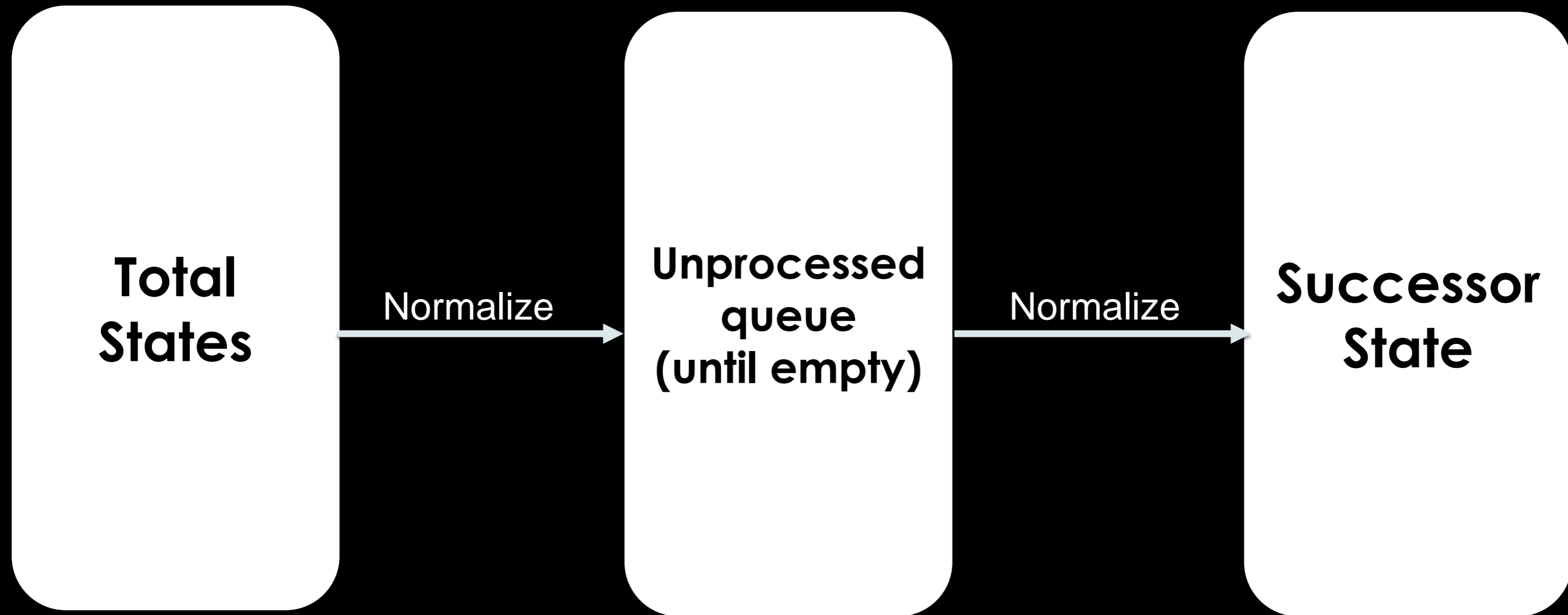
(b) Quotient structure.

# Quotient Transition System:

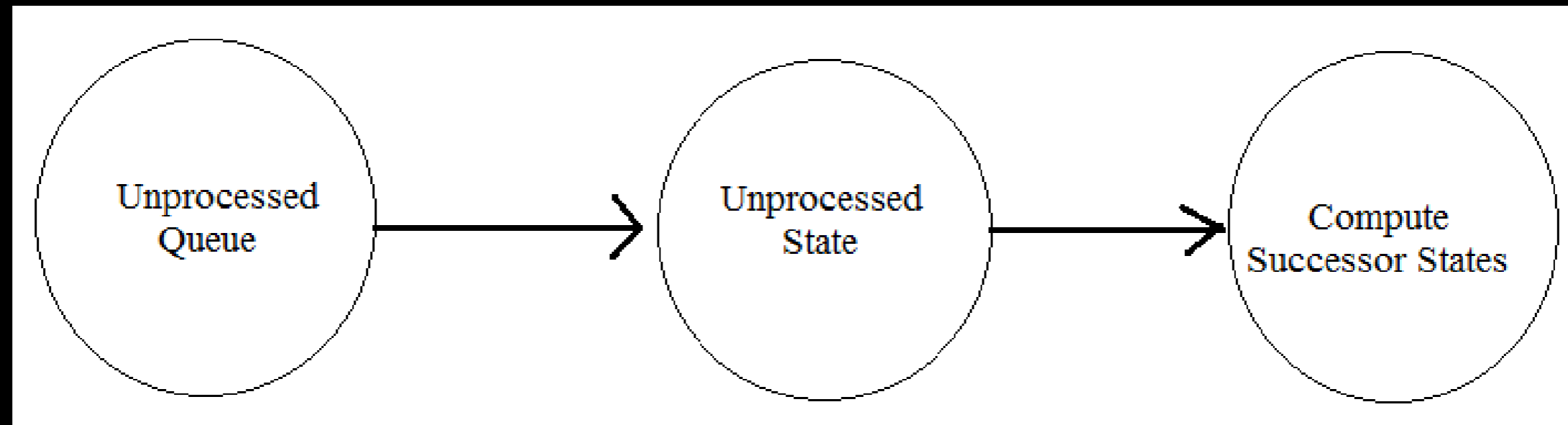
- ▶ Constructing a Quotient Transition System consists of two steps:
  - ▶ 1. The computation of  $S_P$
  - ▶ 2. The computation of  $R_P$  .



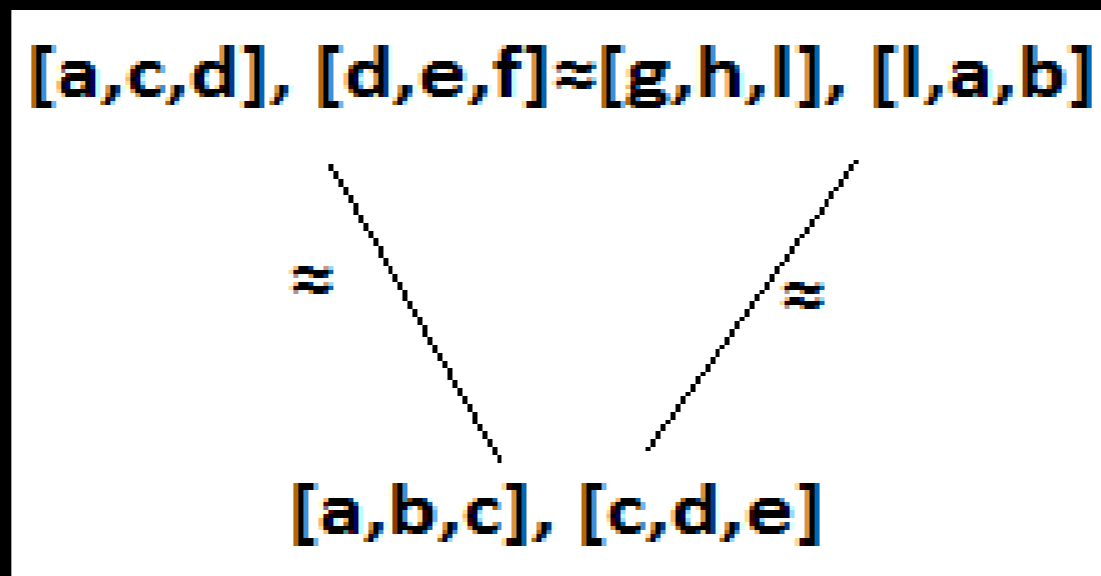
# Quotient Transition System:



# Quotient Transition System:



*Computation of  $S_P$*



*Normalize: Unique representative in the equivalence relation for pairs of states.*

# Quotient Transition System:

- Computation of  $R_p$ :
  - ▶  $\text{NORMALIZE}(\text{update}_p(p, \langle a \rangle); \text{update}_p(q, \langle a \rangle))$  is equal for all  $a$ .
  - ▶ Computing successors under the finite number of accesses.

## Results:

- ▶ Sensitivity results for LRU, FIFO, PLRU, and MRU at associativity ranging from 2 to 8 is obtained.
- ▶ The data obtained are precise sensitive ratios.

# Results:

Policies/Associativity	2	3	4	5	6	7	8
LRU	1,2	1,3	1,4	1,5	1,6	1,7	1,8
FIFO	2,2	3,3	4,4	5,5	6,6	7,7	8,8
PLRU	1,2	-	$\infty$	-	-	$\infty$	-
MRU	1,2	3,4	5,6	7,8	MEM	MEM	MEM

Miss-Sensitivity ratio, k, and additive constant, c, for FIFO, PLRU, and LRU.

**Note:** MEM indicates the algorithm ran out of memory on a 2gb machine.

# Results:

Policies/Associativity	2	3	4	5	6	7	8
LRU	1,2	1,3	1,4	1,5	1,6	1,7	1,8
FIFO	0,0	0,0	0,0	0,0	0,0	0,0	0,0
PLRU	1,2	-	1/3, 5/3	-	-	$\infty$	1/11, 19/11
MRU	1,2	0,0	0,0	0,0	MEM	MEM	MEM

Hit-Sensitivity ratio k, and subtractive constant c, for FIFO, PLRU, and LRU.

**Note:** MEM indicates the algorithm ran out of memory on a 2gb machine.

# Results:

LRU is best replacement policy, most robust.

## Open Question:

When the access sequence is restricted computing precise sensitive ratios become difficult.

Computing a quotient transition system, as done in this paper becomes improbable.



Common Ground:

**ROBUSTNESS**

Thank you!