SAARLAND UNIVERSITY
COMPUTER SCIENCE

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Verification of Real-Time Systems SS 2015

Assignment 2

Deadline: Thursday, May 14, 2015, 14:00 (no lecture)
Please send your submission via e-mail to sebastian.hahn@cs.uni-saarland.de, or bring it to room 402 (in building E1.3) on Wednesday, May 13, 2015, until 16:00.

Exercise 2.1: Partial Order (10 = 3+2+2+3 Points)

Prove or disprove whether the following binary relations are partial orders.

1. Reachability of vertices (reflexive, transitive closure of the edge relation) in an directed, acyclic graph $G = (V, E)$ with vertex set V and edge relation E.
2. Reachability of vertices in a directed graph $G = (V, E)$ with vertex set V and edge relation E.
3. Reachability of vertices in an undirected graph $G = (V, E)$ with vertex set V and edge relation E.
4. For a set X and a partially-ordered set P, the function space $F : X \to P$, where $f \leq g$ if and only if $f(x) \leq g(x)$ for all x in X.

Exercise 2.2: Monotone Functions (13 = 3+4+3+3 Points)

1. Which of the following functions are monotone (under the natural order $\leq$)? Justify your answers.
   - $f : \mathbb{N} \to \mathbb{N}, f(x) = x + 1$
   - $g : \mathbb{N} \to \mathbb{N}, f(x) = 0$
   - $h : \mathbb{N} \to \mathbb{N}, f(x) = 5 \cdot x$
   - $i : \mathbb{N} \to \mathbb{N}, f(x) = x \bmod 5$
   - $j : \mathbb{N} \to \mathbb{N}, f(x) = \lfloor x/5 \rfloor$
2. Given $A = \mathcal{P}(\mathbb{N})$ and $B = \mathbb{N}$, choose partial orders for A and B, and find two functions $f, g : A \to B$ such that f is monotone under the chosen order and g is not.
3. Let $f : P \to Q$ and $g : Q \to R$ be monotone functions. Prove that the function composition $g \circ f : P \to R$ is also monotone.
4. Find an order such that $F(X) = \{f(x) \mid x \in X\}$ is a monotone function. Give a proof for your answer.

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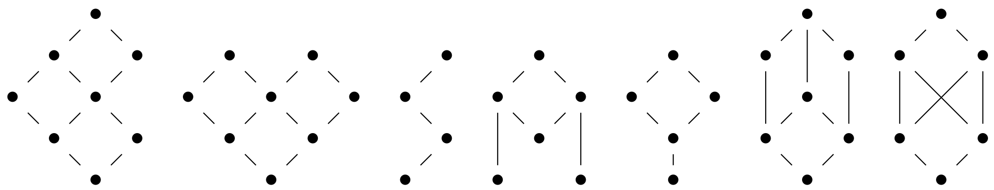
Exercise 2.3: Complete Lattices (9 = 3+3+3 Points)

Let $|$ be the relation of divisibility: $a|b \Leftrightarrow \exists t \in \mathbb{N} \setminus \{0\}, a \cdot t = b$. Which of the following are complete lattices? Justify your answers.

1. $(\mathbb{N}, |)$
2. $(\mathbb{N} \setminus \{0\}, |)$
3. $(\mathbb{N} \setminus \{0\} \cup \{\infty\}, |)$, where $\forall t : t \cdot \infty = \infty \cdot t = \infty$

Exercise 2.4: Complete Lattices (7 Points)

Which of the following Hasse diagrams represent complete lattices?



(a) (b) (c) (d) (e) (f) (g)