Design and Analysis of Real-Time Systems SS 2013

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Assignment 7 - Solutions

Problem 1: Must Analysis

We assume that M denotes the set of memory blocks, and ϵ an empty cache line.

1. $C := \{c \in (M \cup \{\epsilon\})^k \mid c = [c_0, \ldots, c_{k-1}] \wedge \forall m \in M.(m = c_i \wedge m = c_j \Rightarrow i = j)\}$. In a state $c = [c_0, \ldots, c_{k-1}] \in C$, c_0 is the most-recently-used element, and c_{k-1} the least-recently-used element.
2. $A := \{a \in (\mathcal{P}(M))^k \mid a = [a_0, \ldots, a_{k-1}] \wedge (\forall i, j.i \neq j \Rightarrow a_i \cap a_j = \emptyset)\}$

In the following, we will use two helper functions, $age : C \times M \to \mathbb{N} \cup \{\infty\}$ and $age_{abs} : A \times M \to \mathbb{N} \cup \{\infty\}$, that compute the age of a memory block. They are defined as follows:

$$age([c_0, \ldots c_{k-1}], m) := \begin{cases} i & \text{if } m = c_i \\ \infty & \text{otherwise} \end{cases}$$

$$age_{abs}([a_0, \ldots a_{k-1}], m) := \begin{cases} i & \text{if } m \in a_i \\ \infty & \text{otherwise} \end{cases}$$

3. $\gamma(a) := \{c \in C \mid \forall m \in M.age(c, m) \leq age_{abs}(a, m)\}$
4. $\alpha(S) := [a_0, \ldots, a_{k-1}]$, where $m \in a_i$ iff $i = \max_{c \in S} age(c, m)$
5. $a \sqsubseteq_A b := \forall m \in M.(age_{abs}(a, m) \leq age_{abs}(b, m))$
6. $a \sqcup b := [x_0, \ldots, x_{k-1}]$, where $m \in x_i$ iff $i = \max\{age_{abs}(a, m), age_{abs}(b, m)\}$
7. $$access([c_0, \ldots c_{k-1}], m) := \begin{cases} [c_i, c_0, \ldots c_{i-1}, c_{i+1}, \ldots, c_{k-1}] & \text{if } m = c_i \\ [m, c_0, \ldots c_{k-2}] & \text{otherwise} \end{cases}$$
8. $$access_{abs}([a_0, \ldots, a_{k-1}], m) := \begin{cases} [\{m\}, a_0, \ldots, a_{i-1} \cup a_i \setminus \{m\}, a_{i+1}, \ldots, a_{k-1}] & \text{if } m \in a_i \\ [\{m\}, a_0, \ldots a_{k-2}] & \text{otherwise} \end{cases}$$