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Deadline: May 2, 2013, before the lecture

Assignment 1

Problem 1: Real-Time Systems (2+2+2 Points)

1. What are the differences between hard and soft real-time systems?
2. Find two examples for each, and describe possible consequences if these systems miss their deadlines.
3. Describe two reasons why the actual “worst-case execution time” is typically higher than the “maximal observed execution time”.

Problem 2: Monotone Functions (3+4+3 Points)

1. Which of the following functions are monotone (under the natural order)? Justify your answers.
 - $f : \mathbb{N} \rightarrow \mathbb{N}, f(x) = x + 1$
 - $g : \mathbb{N} \rightarrow \mathbb{N}, f(x) = 0$
 - $h : \mathbb{N} \rightarrow \mathbb{N}, f(x) = 5 \cdot x$
 - $i : \mathbb{N} \rightarrow \mathbb{N}, f(x) = x \bmod 5$
 - $j : \mathbb{N} \rightarrow \mathbb{N}, f(x) = \lfloor x/5 \rfloor$
2. Given $A = \mathcal{P}(\mathbb{N})$ and $B = \mathbb{N}$, choose partial orders for A and B , and find two functions $f, g : A \rightarrow B$ such that f is monotone under the chosen order and g is not.
3. Let $f : P \rightarrow Q$ and $g : Q \rightarrow R$ be monotone functions. Prove that the function $g \circ f : P \rightarrow R$ is also monotone.

Problem 3: Complete Lattices (3+3+3 Points)

Let $|$ be the relation of divisibility: $a|b \Leftrightarrow \exists t, at = b$. Which of the following are complete lattices? Justify your answers.

1. $(\mathbb{N}, |)$
2. $(\mathbb{N} \setminus \{0\}, |)$
3. $(\mathbb{N} \setminus \{0\} \cup \{\infty\}, |)$, where $\forall t : t \cdot \infty = \infty \cdot t = \infty$

Problem 4: Interval Analysis (5 Points)

You are given the following program:

```
R1 = R1 % 6
R2 = 1
R3 = 1
R4 = 1
while (R1 > 2) do (
  R4 = R2 + R3
  R2 = R3
  R3 = R4
  R1 = R1 - 1
)
return R4
```

Build the corresponding control-flow graph, determine a loop bound and perform an interval analysis by hand.