Abstract Interpretation of FIFO Replacement

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Outline

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Notions in Timing Analysis

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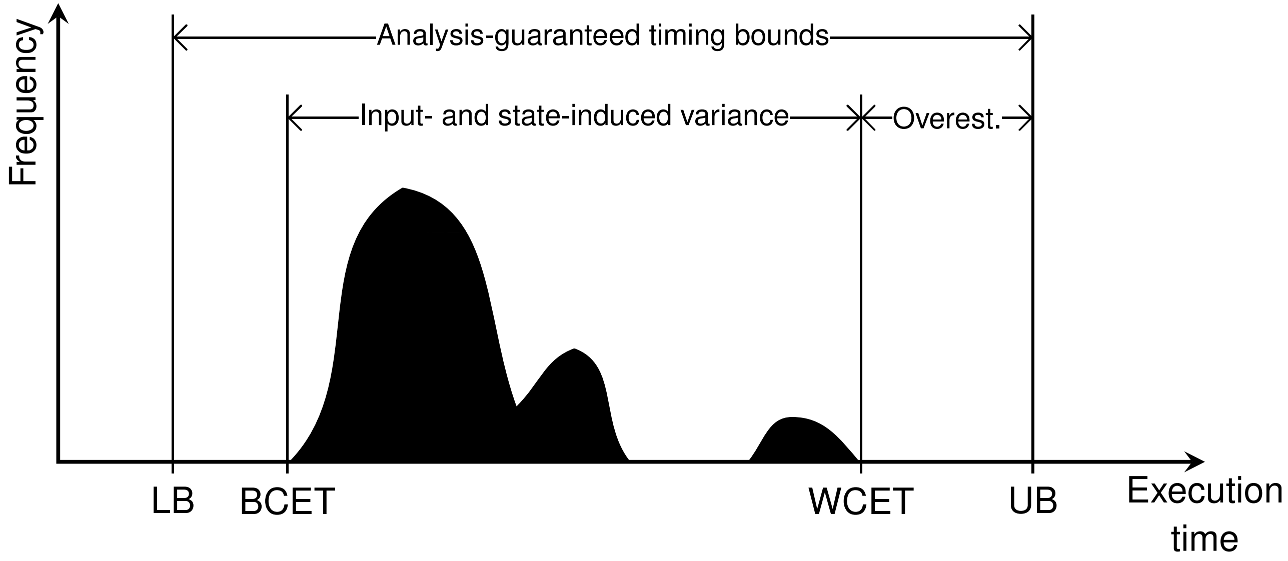

- Execution time depends on
  - program input
  - initial hardware state
- Bounds required for schedulability analysis of real-time systems

Static Timing-Analysis Framework

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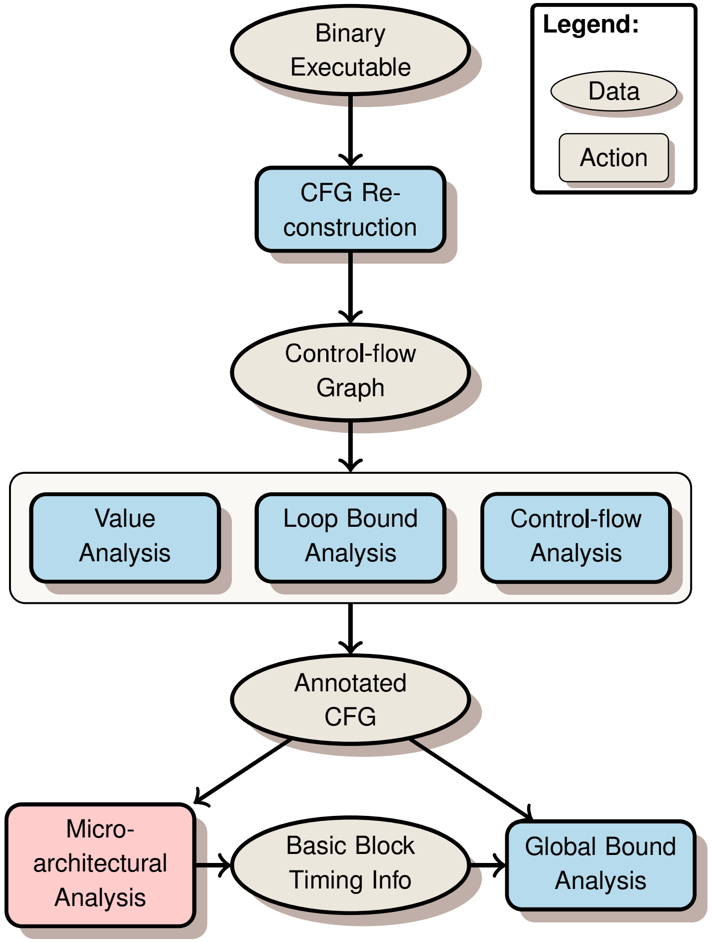
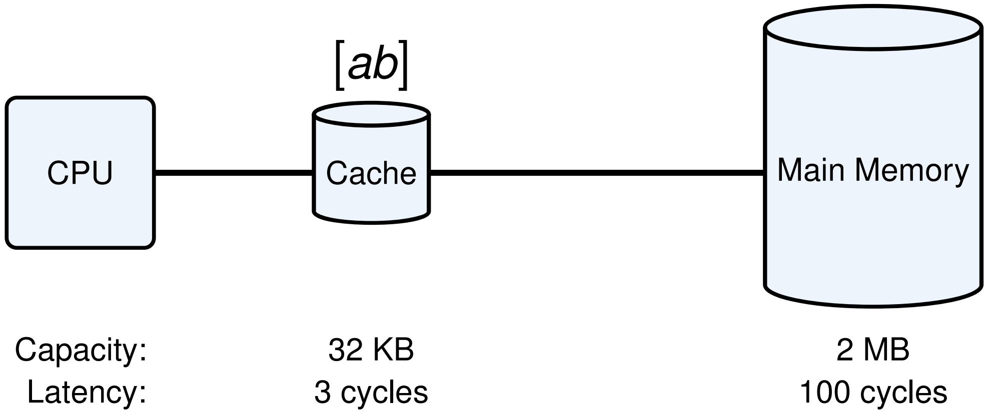


Framework implemented by *aiT* of AbsInt

Micro-architectural analysis

- models pipeline, caches, buses, etc.
- derives bounds on BB exec. times
- is an abstract interpretation with a huge domain
- is the computationally most expensive module

Caches and Replacement Policies



- Caches transparently buffer memory blocks

Caches and Replacement Policies

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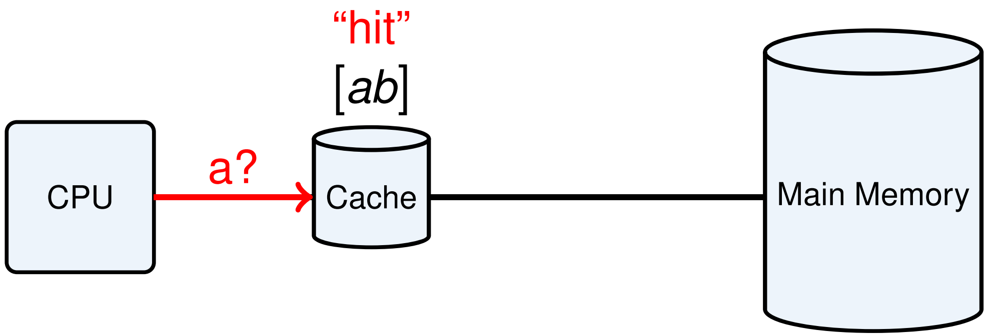

Capacity:	32 KB	2 MB
Latency:	3 cycles	100 cycles

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Caches and Replacement Policies

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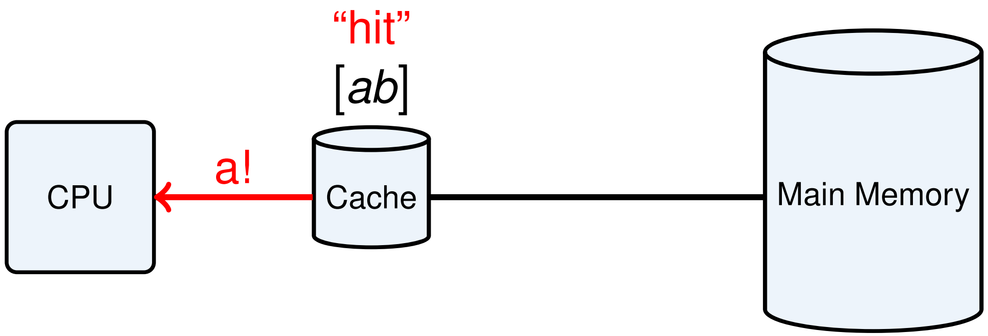

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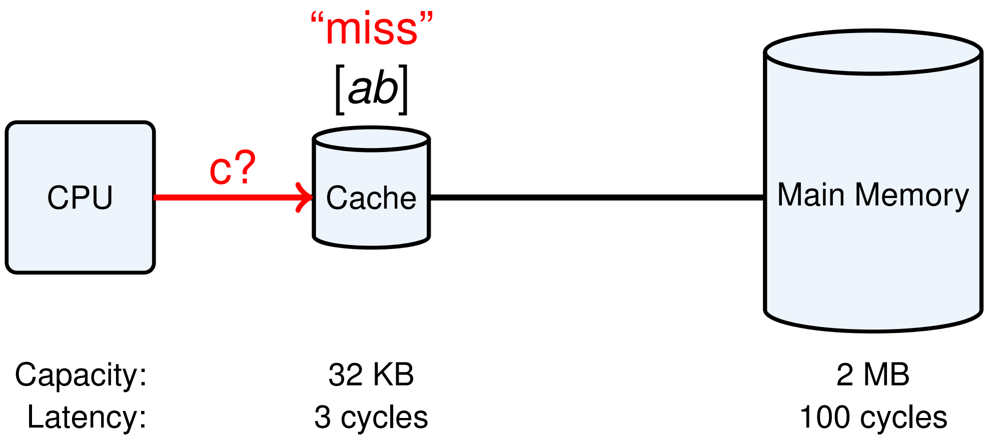

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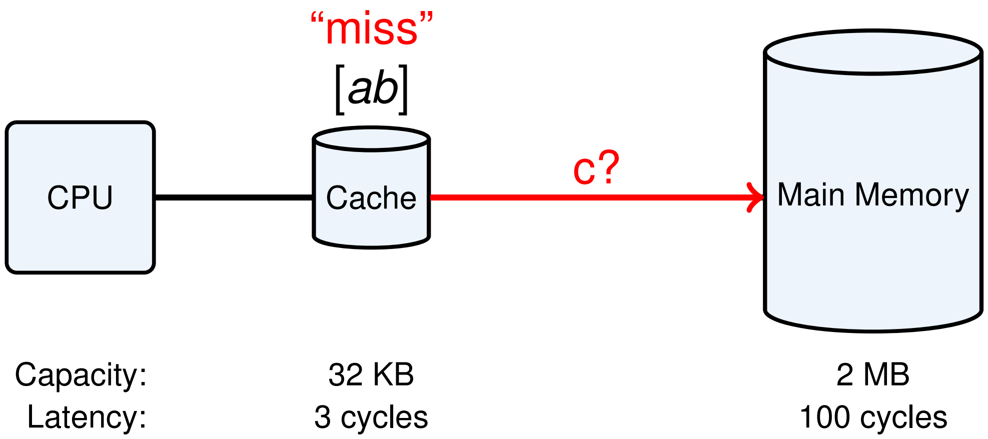

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Caches and Replacement Policies

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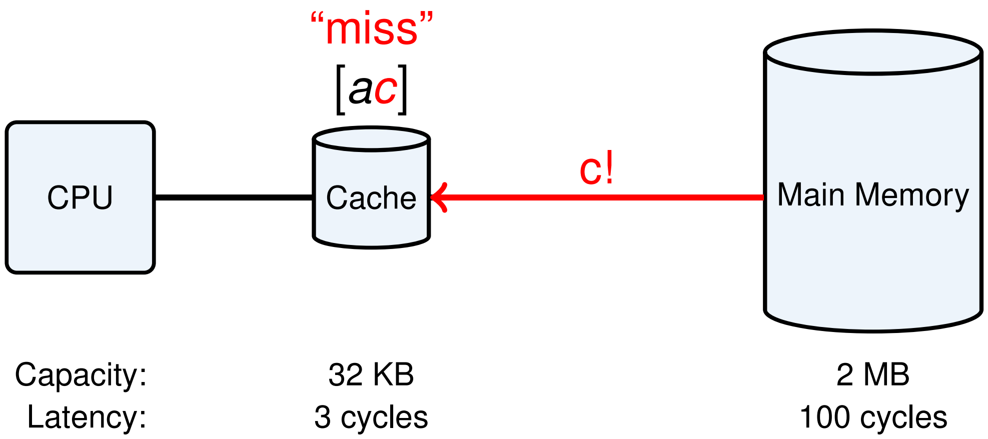
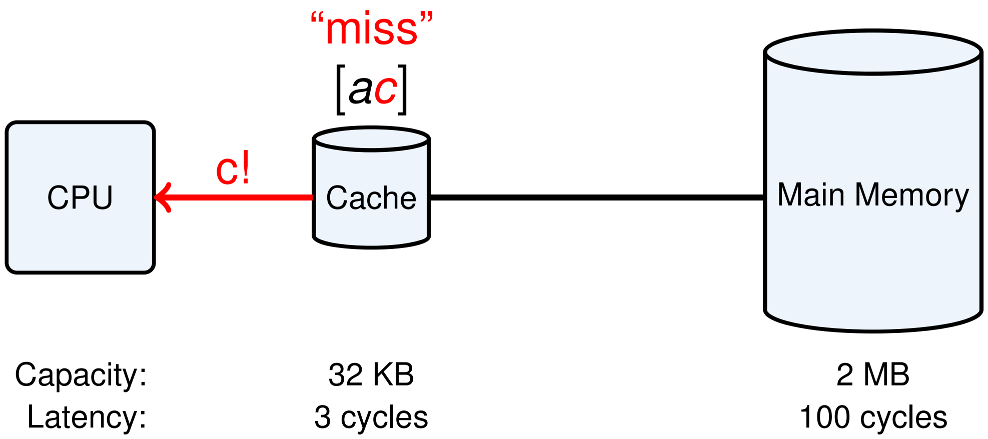


- Caches transparently buffer memory blocks
- Replacement policy *dynamically* decides which element to replace

LRU	least recently used
PLRU	pseudo LRU
FIFO	first-in first-out

- Have great influence on abstraction and (obtainable) analysis precision

Caches and Replacement Policies



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Cache Analysis: Motivation & Application

- Cache performance has great influence on overall performance
- Need tight bounds on cache performance
- Otherwise derived timing bounds may be useless:
  - tasks are deemed not schedulable
  - waste of hardware resources

- Application: Buffers with transparent replacement
  - Instruction- and data-caches
  - Branch target buffers (BTB, BTIC)
  - Translation lookaside buffers (TLB)

Static Cache Analysis

- derives approximations to cache contents at each program point
- in order to classify memory accesses as cache hits or cache misses

Must-information

- Underapproximation of cache contents
- Used to soundly classify cache hits

May-information

- Overapproximation of cache contents
- Used to soundly classify cache misses

Outline

1. Introduction & Motivation
   - Timing Analysis
   - Cache Analysis
2. Abstract Interpretation of FIFO
   - Challenge FIFO Replacement
   - Domain Cooperation
   - Must Analysis
   - May Analysis
3. Evaluation
   - Related Work
   - Analysis Precision
4. Summary

Concrete Semantics: What is FIFO?

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- State of FIFO cache of size k: $s \in \mathcal{S} := \mathcal{T}^k$

$$\overset{\text{last-in}}{\downarrow}\qquad\overset{\text{first-in}}{\downarrow}$$
$$[t_0, \ldots, t_{k-1}]$$

- Examples:

$$[d, c, b, a] \xrightarrow[hit]{c} [d, c, b, a]$$

$$[d, c, b, a] \xrightarrow[miss]{e} [e, d, c, b]$$

- Update: $\mathcal{U}_\mathcal{S} : \mathcal{S} \times \mathcal{T} \rightarrow \mathcal{S}$

$$\mathcal{U}_\mathcal{S}([t_0, \ldots, t_{k-1}], t) := \begin{cases} [t_0, \ldots, t_{k-1}] & : \exists i : t = t_i \quad \text{“cache hit”} \\ [t, t_0, \ldots, t_{k-2}] & : \text{otherwise} \quad \text{“cache miss”} \end{cases}$$

Challenge: How to Predict Hits?

- Consider a FIFO cache with unknown contents

$$[?, ?, ?, ?] \xrightarrow[hit]{a} [?, a, ?, ?] \xrightarrow[hit]{b} [?, a, ?, b]$$

$$[?, ?, ?, ?] \xrightarrow[hit]{a} [?, ?, ?, a] \xrightarrow[miss]{b} [b, ?, ?, ?]$$

- If *a may* be a hit, then *b may* evict *a*

$\Rightarrow$ Can only predict hits for most recently accessed element

- Can one do better?

$$[?, ?, ?, ?] \xrightarrow[miss]{a} [a, ?, ?, ?] \xrightarrow[miss]{b} [b, a, ?, ?]$$

- If *a is* a miss, then *b cannot* evict *a*

$\Rightarrow$ Can predict hits for *a* until *k* further misses might have happened

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- If *a is* a miss, then *b cannot* evict *a*

⇒ Can predict hits for *a* until *k* further misses might have happened

⇒ Need may-information to obtain precise must-information

A Solution. Well, our Contributions

- Framework for static cache analysis
  - policy independent
  - couples must- and may-analyses
  - analyses cooperate via “update reduction”
- FIFO must-analysis
  - can profit from may-information
  - hence, also better must-information
- FIFO may-analysis
  - utilizes order of hits and misses
  - more precise than prior analyses

Framework and Classification

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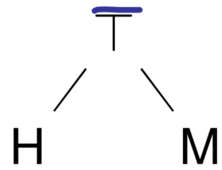
Domain: $\mathit{Fifo} := \mathit{Must} \times \mathit{May}$

Classification: $\mathit{Class} := \{\mathrm{H}, \mathrm{M}\}^{\top}$

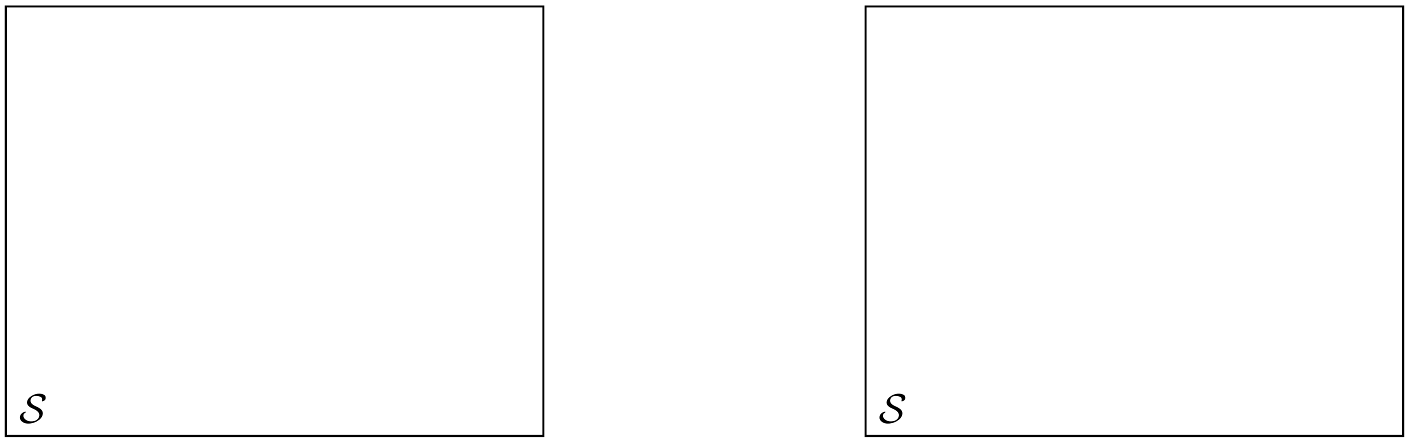


H : cache hit
M : cache miss
$\top$: unclassified

$$\mathcal{C}_{\mathit{Fifo}} : \mathit{Fifo} \times \mathcal{T} \longrightarrow \mathit{Class}$$
$$\mathcal{C}_{\mathit{Fifo}}((\mathit{must}, \mathit{may}), t) := \mathcal{C}_{\mathit{Must}}(\mathit{must}, t) \sqcap \mathcal{C}_{\mathit{May}}(\mathit{may}, t)$$

Domain Cooperation via Update Reduction

$$(\mathit{must}, \mathit{may}) \xrightarrow{\text{Independent}} (\mathit{must}', \mathit{may}')$$



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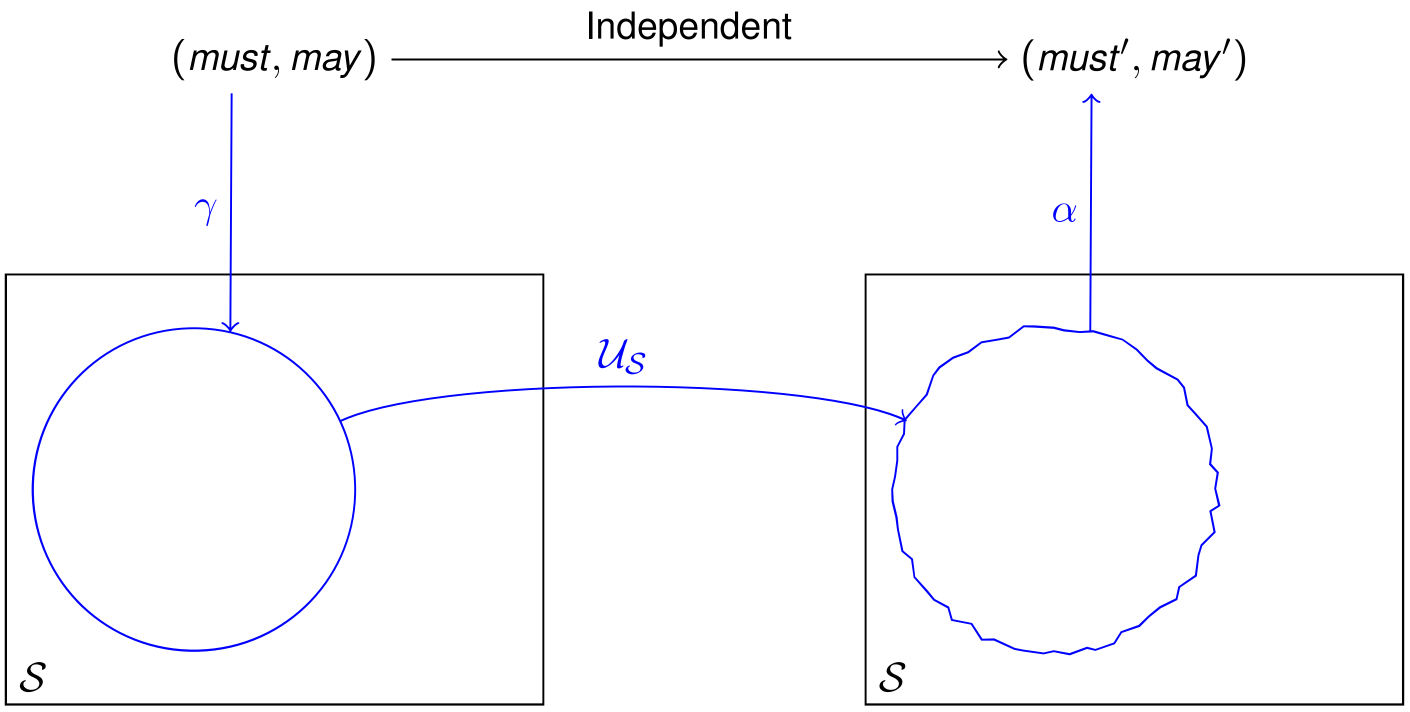

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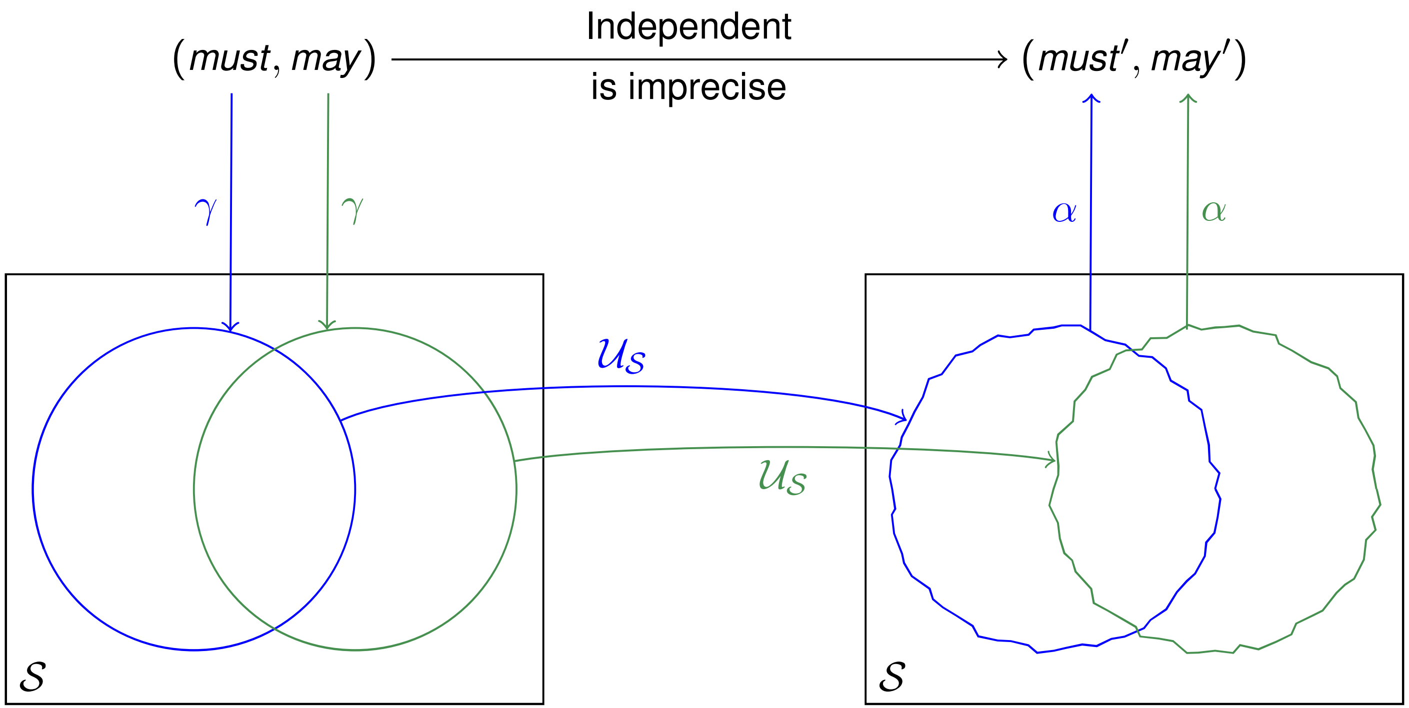
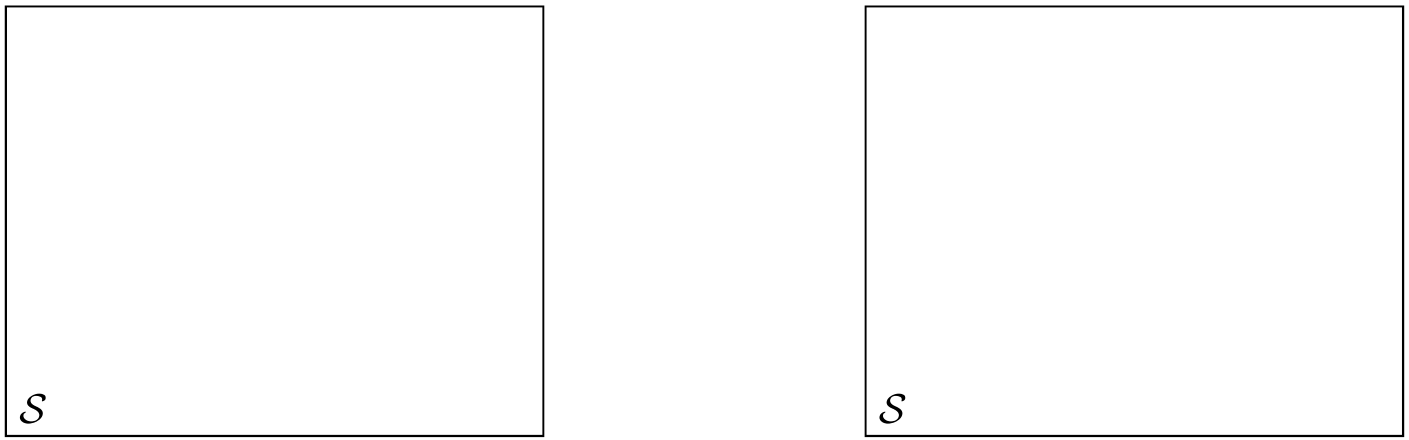


Domain Cooperation via Update Reduction

$$(must, may) \xrightarrow{\text{Best possible}} (must', may')$$



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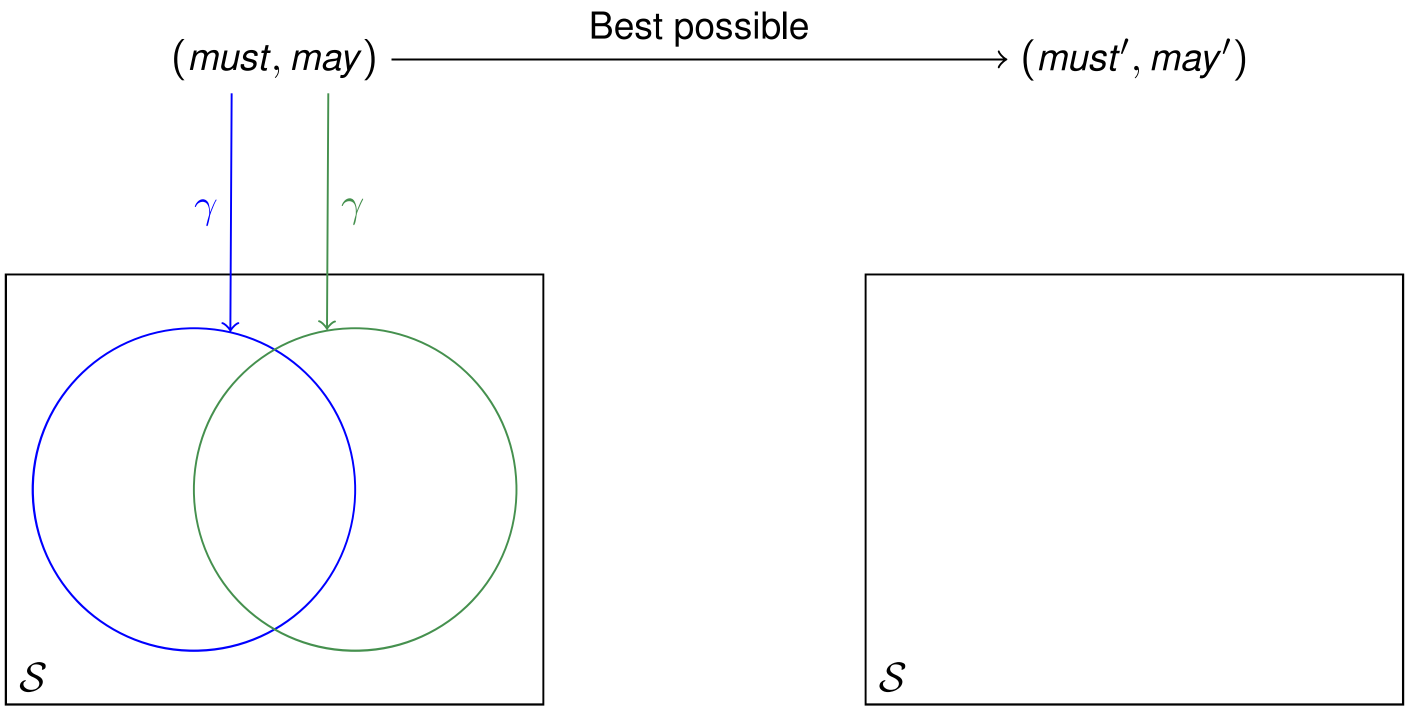

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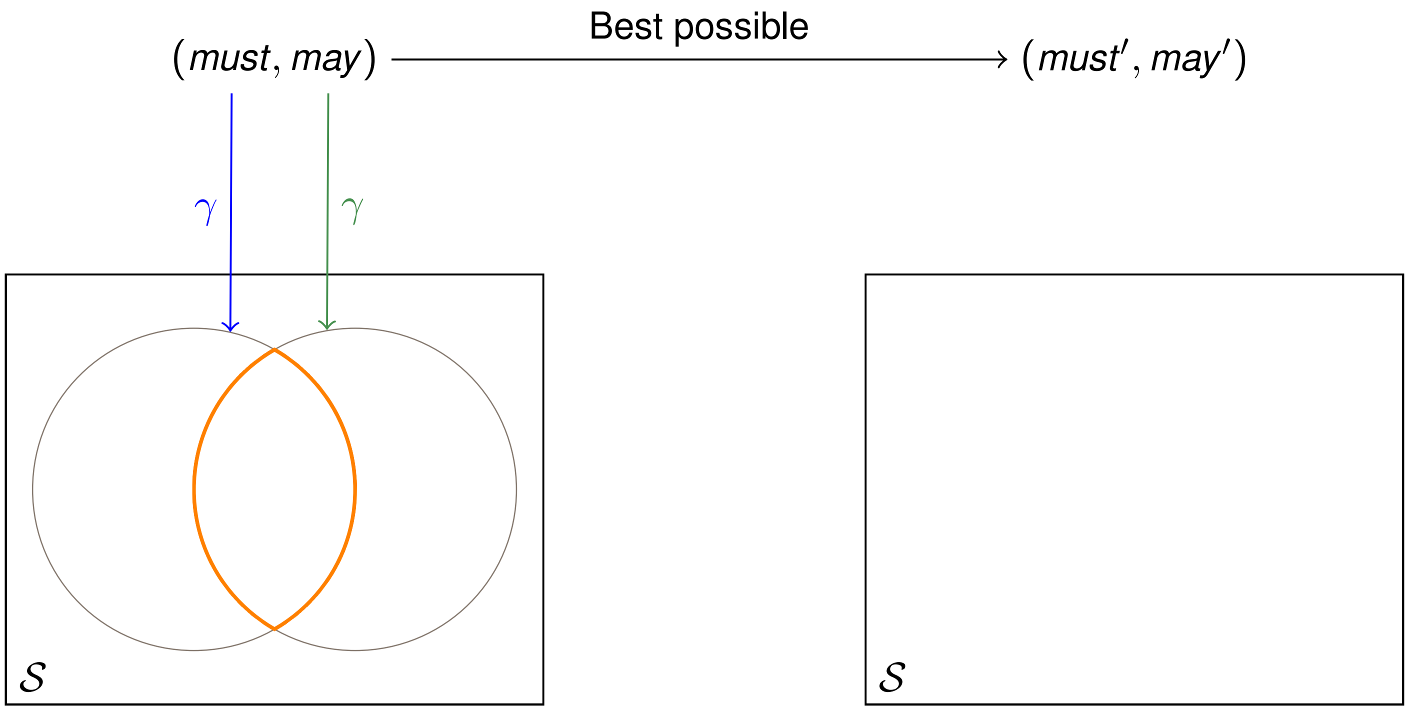

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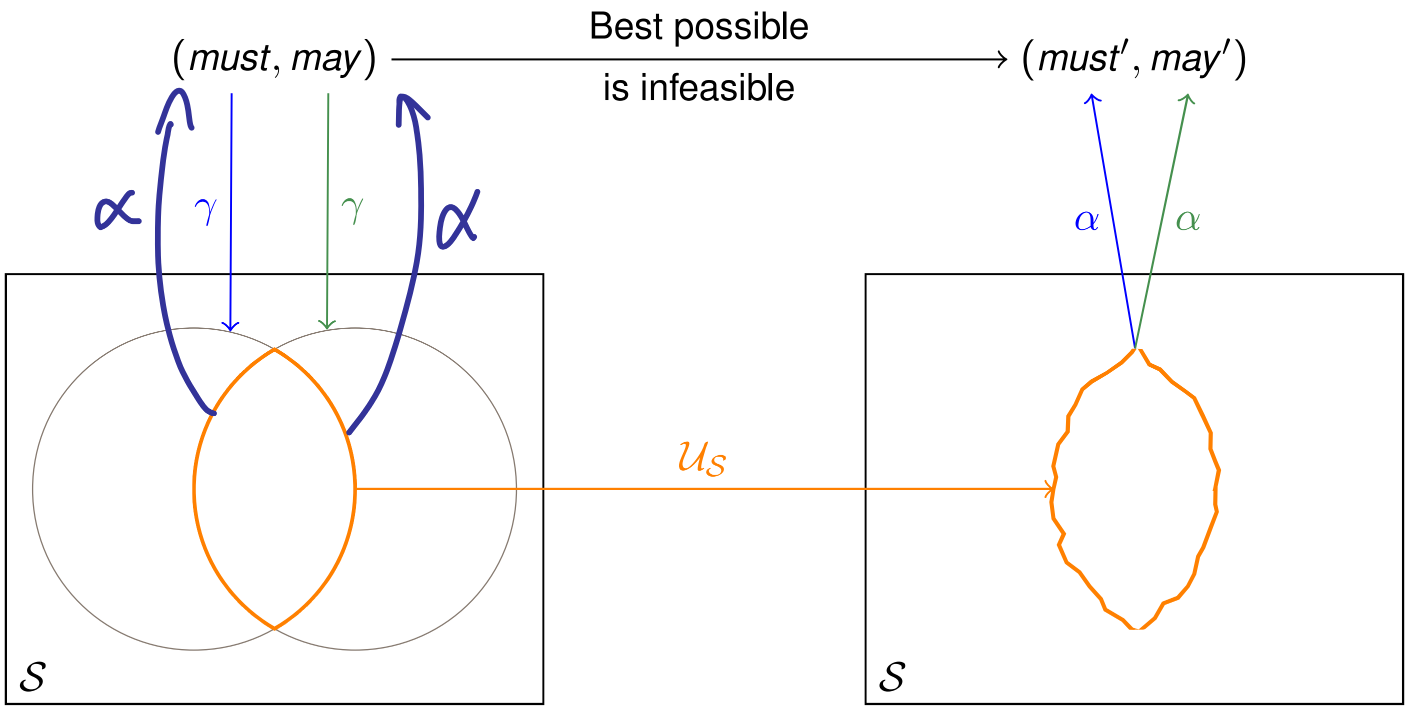

Domain Cooperation via Update Reduction

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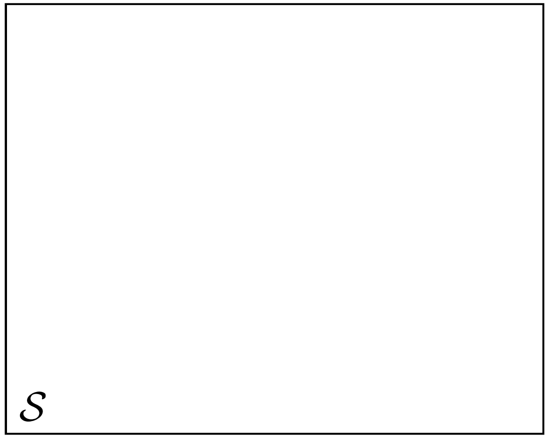
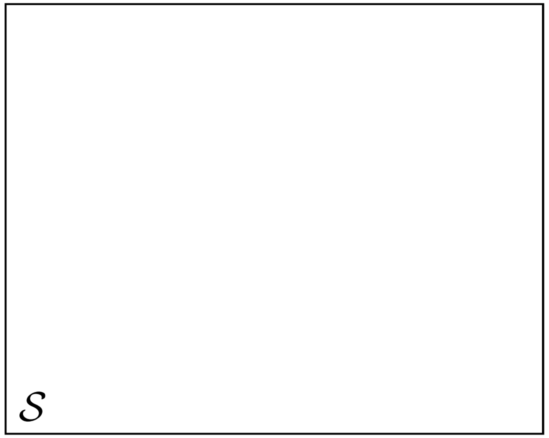
$$(\mathit{must}, \mathit{may}) \xrightarrow[\mathcal{C}_{\mathit{Fifo}}((\mathit{must}, \mathit{may}), t) = \textcolor{red}{\mathrm{M}}]{\text{Using classification}} (\mathit{must}', \mathit{may}')$$





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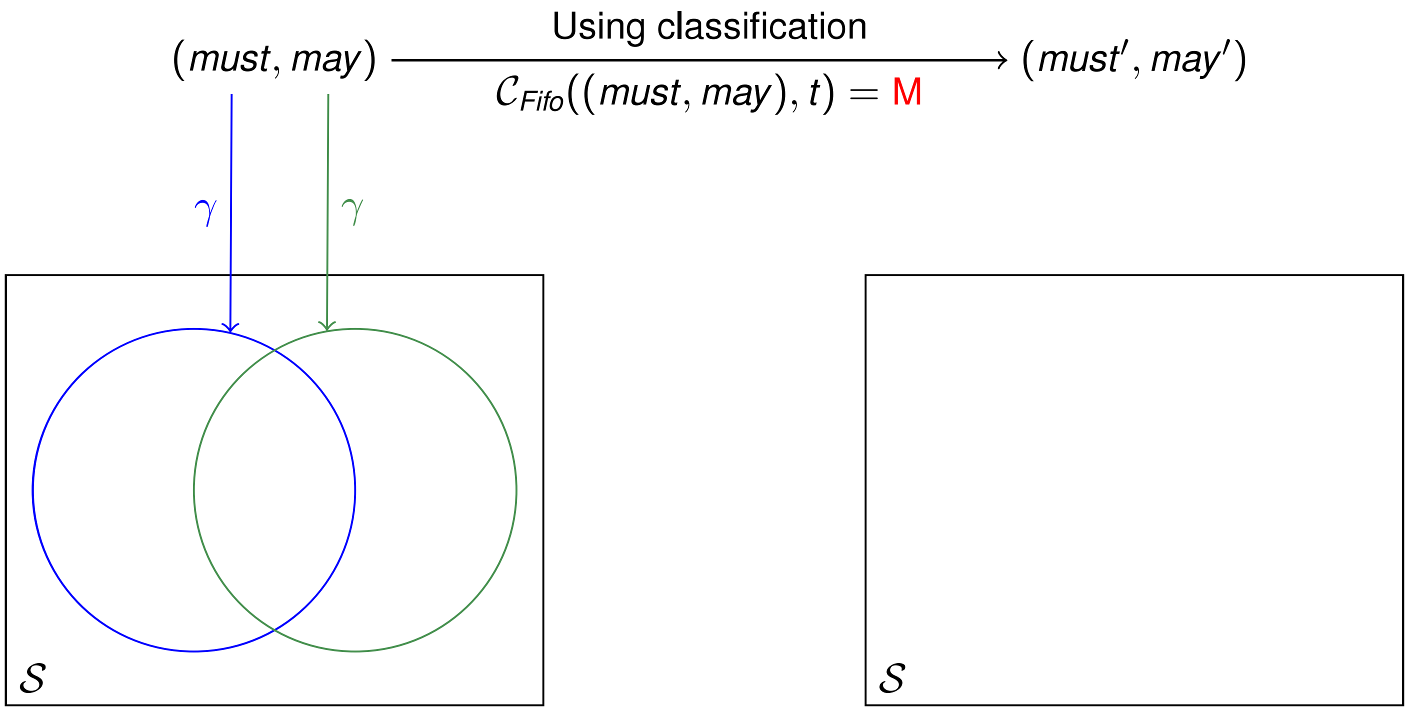

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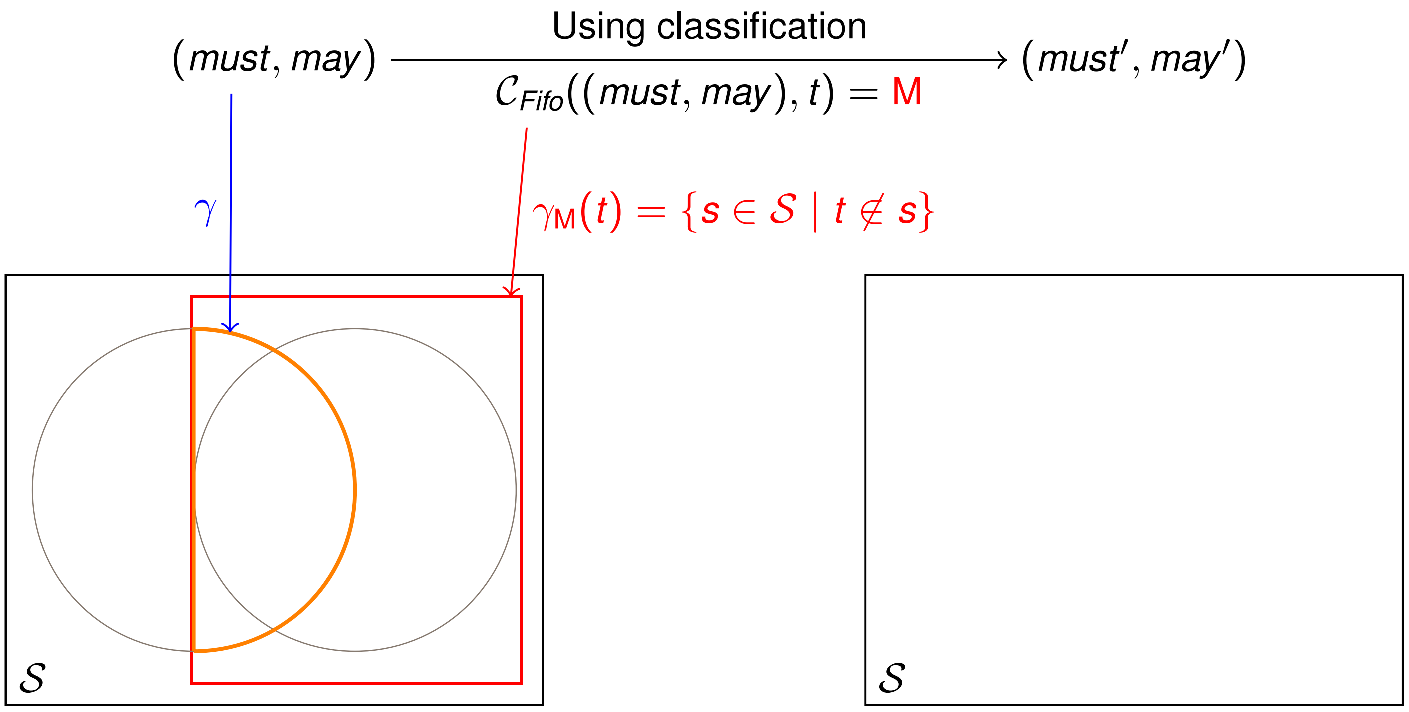

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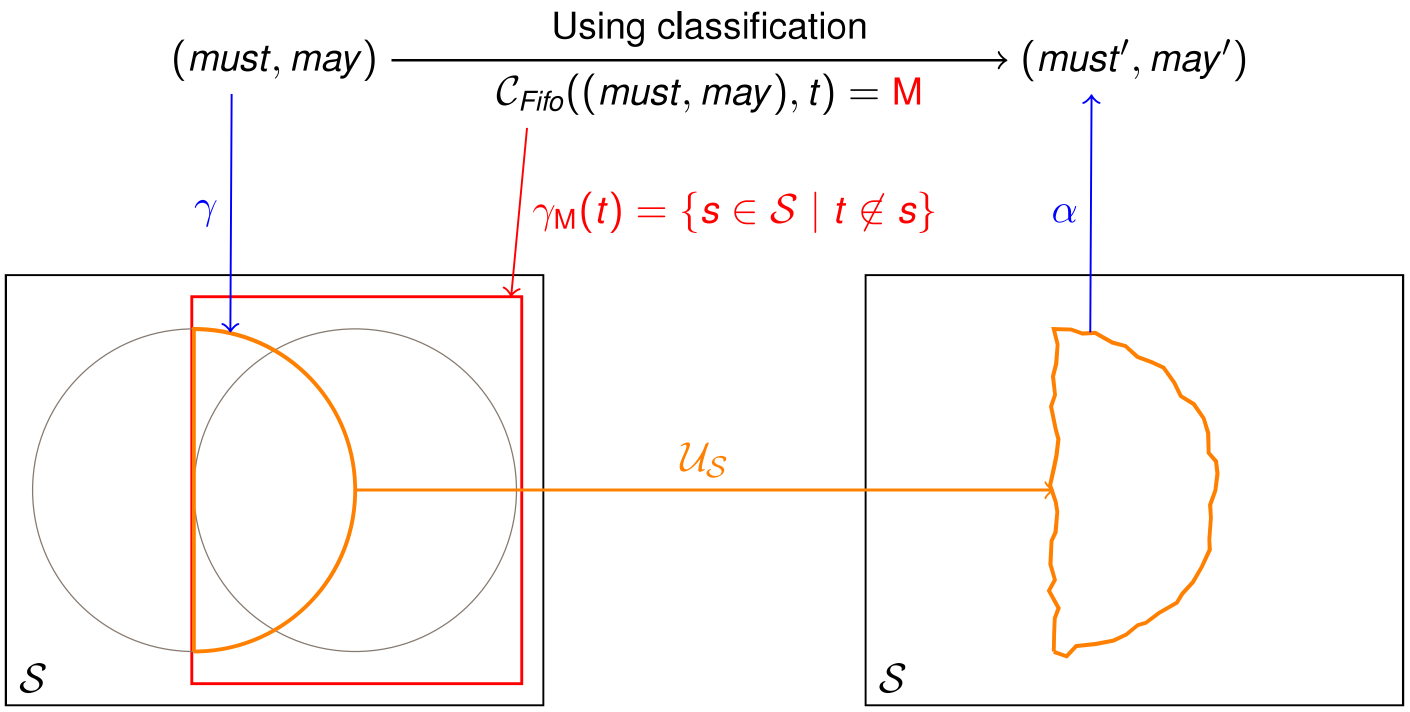

Outline

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Must-Analysis: Potential Misses

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- For FIFO, a newly inserted element is evicted after k misses

$\Rightarrow$ Maintain upper bound on number of misses: Potential misses

- Abstract must-domain closely resembles the concrete domain

$$Must_{Fifo_k} := [T_0, \ldots, T_{k-1}],$$

where $T_i \in \mathcal{P}(\mathcal{T})$, $T_i \cap T_j = \emptyset$, and $\sum |T_i| \leq k$.

- $t \in T_i \Rightarrow$ at most i misses since insertion of t
- Concretization example

$$\gamma([\{f\}, \emptyset, \{a, c\}, \{b\}]) = \{[f, c, a, b], [f, a, c, b]\}$$

Must-Analysis: Update

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$$\mathcal{U}_{Must} : Must \times \mathcal{T} \times Class \rightarrow Must$$

$$\mathcal{U}_{Must}([T_0, \ldots, T_{k-1}], t, cl) := \begin{cases} [\emptyset, T_0, \ldots, T_{k-2} \cup \{t\}] & : cl = \top \\ [T_0, \ldots, T_{k-1}] & : cl = \mathrm{H} \\ [\{t\}, T_0, \ldots, T_{k-2}] & : cl = \mathrm{M} \end{cases}$$

- Misses classified by may-analysis
- Last case only possible due to domain cooperation

Outline

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May-Analysis: Definite Misses

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- How to predict misses?

$\Rightarrow$ Maintain lower bound on number of misses: Definite misses

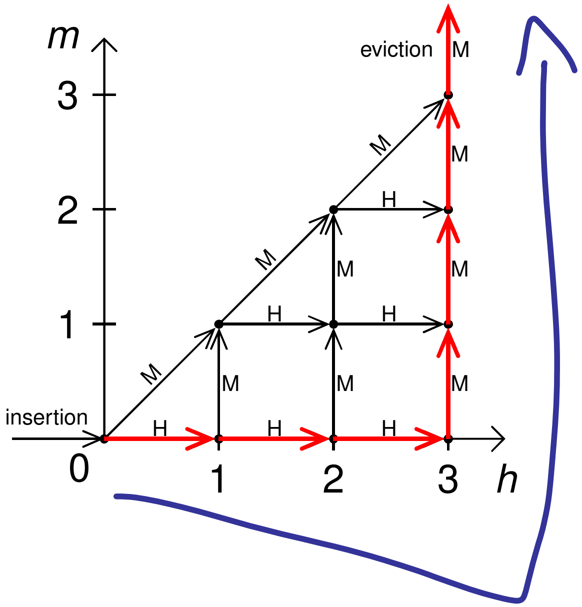
- Initially, anything might be cached
- To classify a miss for an *individual* access, one needs to predict k other misses first

Lemma

A newly inserted element is evicted after accesses to at most $2k - 1$ pairwise different elements.

May-Analysis: Idea “Early Misses”

- Initial state: $[x, c, b, a]$
- Sequences of different length:
  - $\langle a, b, c, e, f, g, h \rangle$
- Common final state: $[h, g, f, e]$



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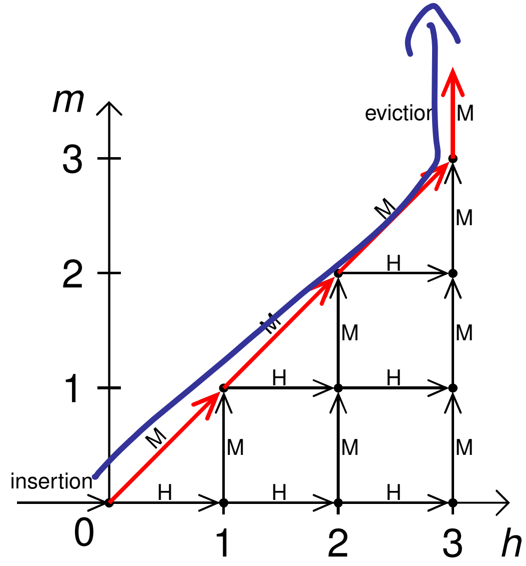

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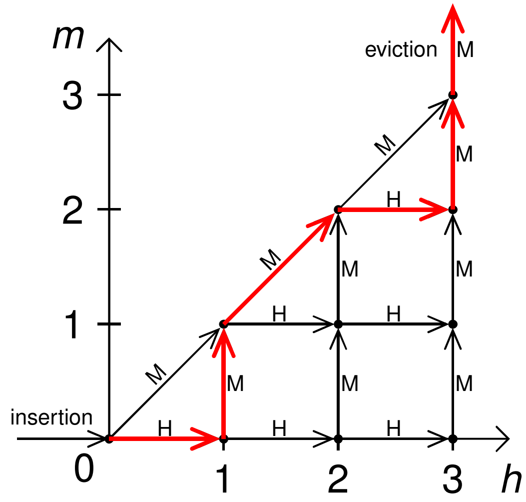

May-Analysis: Idea “Early Misses”

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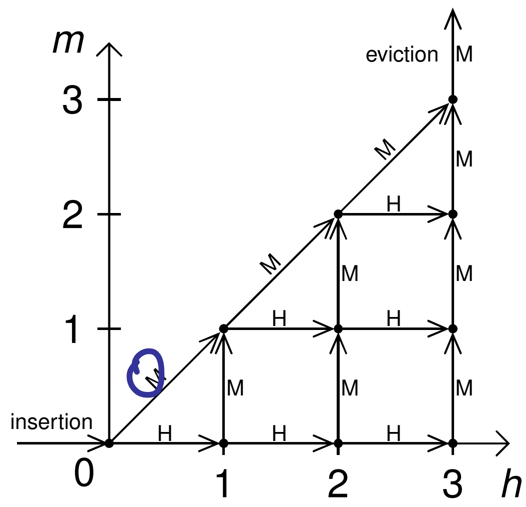
$\Rightarrow$ Sequences differ in number of hits



May-Analysis: Idea "Early Misses"

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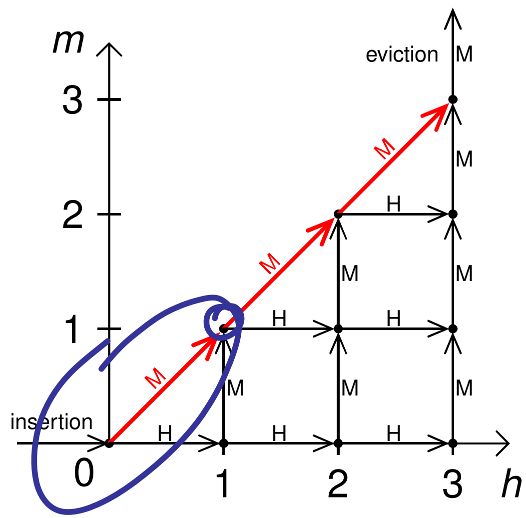

- "Early misses"
  - preclude hits to thereby evicted elements
  - reduce number of possible accesses between insertion and eviction

$\Rightarrow$ Order of hits and misses is important

May-Analysis: Domain

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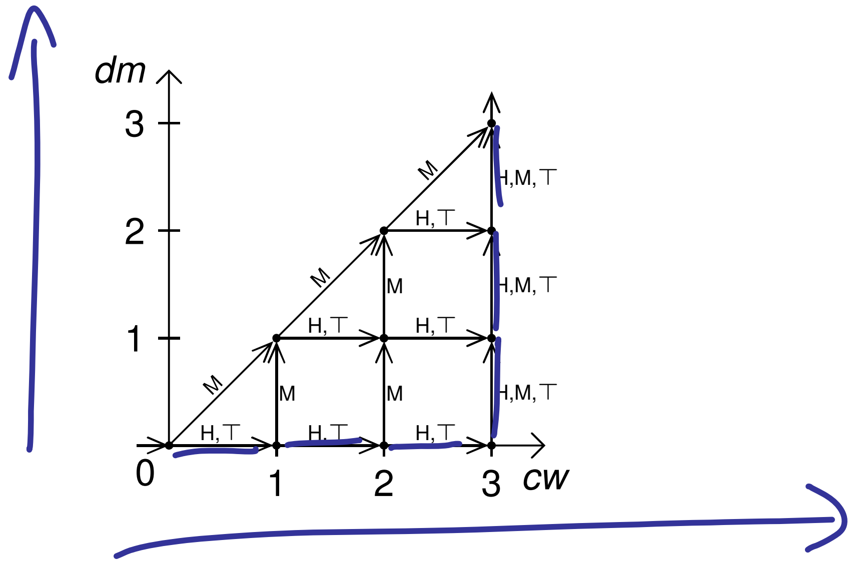
- May analysis approximates position in triangle
- Unclassified access → "take the longer way"



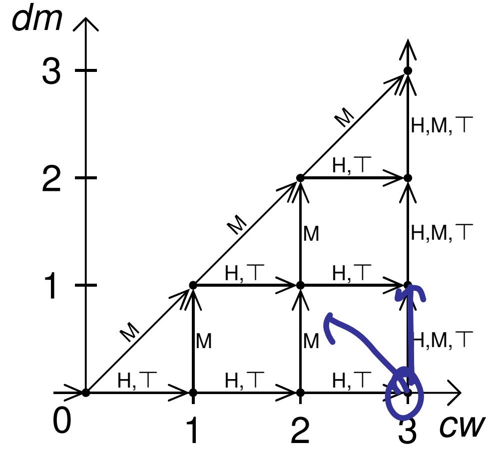
- For each element t, the analysis maintains:

A Set of *Potentially Accessed Elements*

dm Number of *Definite Misses*

cw Number of *Covered Ways* (Covered Cache Positions)

May-Analysis: Example



- Assume sequence $\langle x, a, b, c \rangle$ and all accesses are unclassified
- Then for x one has:

$A = \{a, b, c\}$	a, b and c might have been accessed since the last insertion of x
$dm = 0$	0 misses have definitely happened since the last insertion of x
$cw = 3$	Assuming that all unclassified accesses were hits, then 3 elements of A must be cached

- Consider next access to d

Outline

Relative Competitiveness

- How many misses would FIFO have if LRU has m_{LRU} misses?

Definition: Relative Competitiveness

Policy P is (f, c) miss-competitive relative to policy Q if

$$m_P(p, s) \leq f \cdot m_Q(q, s) + c$$

for all access sequences s and compatible cache states p, q.

- E.g. LRU$(2k - 1)$ is $(1, 0)$ miss-competitive vs. FIFO(k)

$\Rightarrow$ LRU$(2k - 1)$ may-analysis can be used for FIFO(k) may analysis

Evaluation Setup

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- Must-analysis:

  CM Canonical must-analysis (this paper)

- May-analyses:

  No None

  RC Based on relative competitiveness

  EMX Early Miss eXploitation (this paper)

- Instantiations of cache analysis framework:
  - No+CM
  - RC+CM
  - EMX+CM
- Synthetic benchmarks:
  - Random access sequences and program fragments

Evaluation Results

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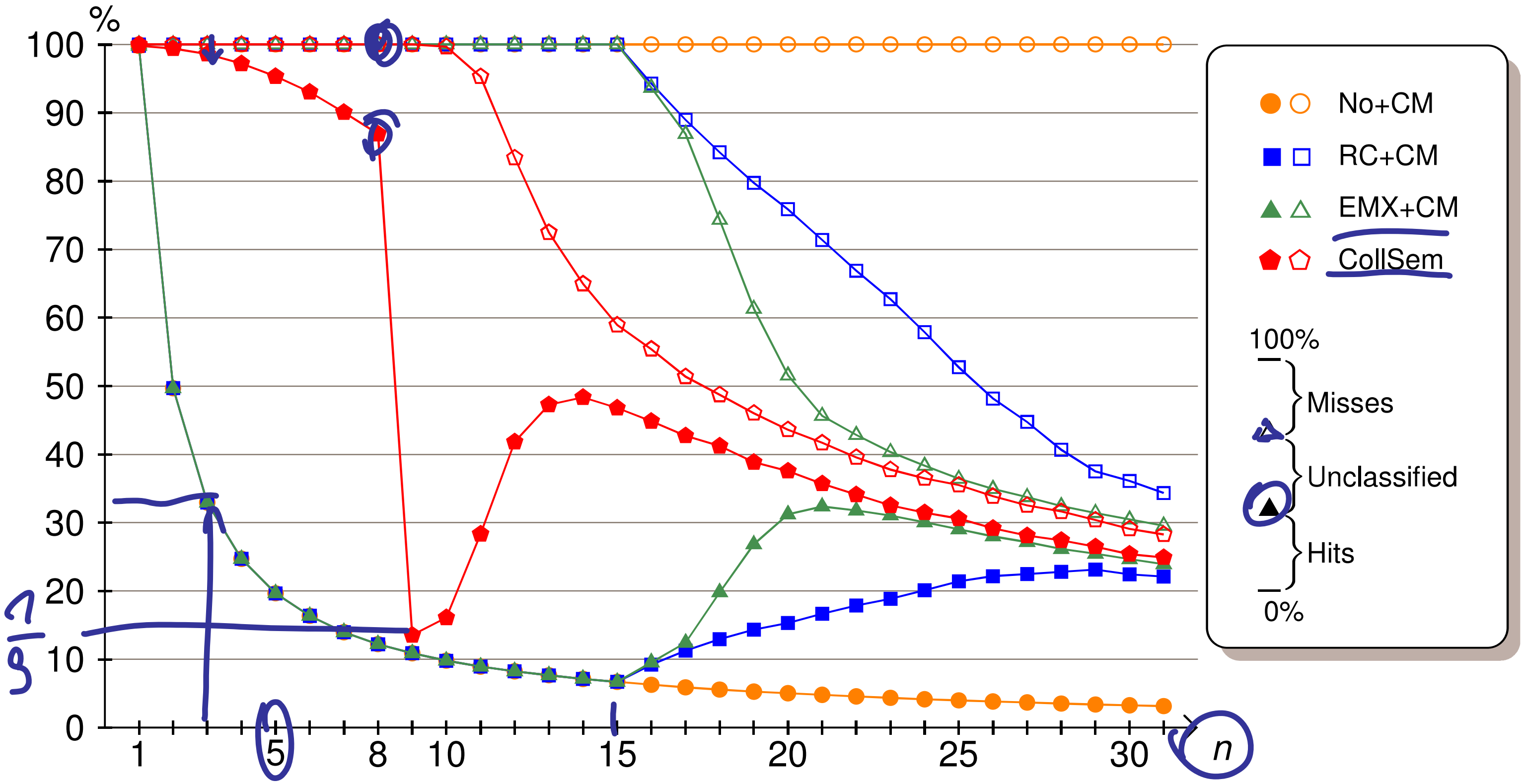
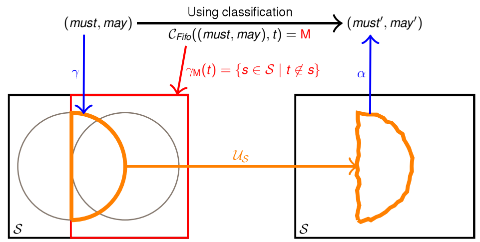


- Average guaranteed hit- and miss-rates for a cache of size 8
- *n* is number of pairwise different elements that are accessed

Outline

Summary

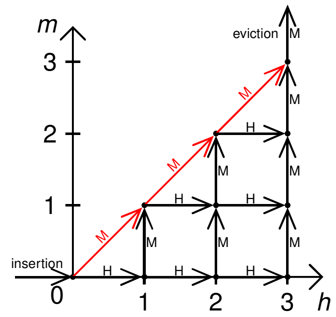
- Cache analysis framework
  - Couple several analyses
  - Cooperation via classifications



- Canonical FIFO must-analysis
  - Potential misses

$$\mathcal{U}_{Must}([T_0, \ldots, T_{k-1}], t, cl) := \begin{cases} [\emptyset, T_0, \ldots, T_{k-2} \cup \{t\}] & : cl = \top \\ [T_0, \ldots, T_{k-1}] & : cl = \mathrm{H} \\ [\{t\}, T_0, \ldots, T_{k-2}] & : cl = \mathrm{M} \end{cases}$$

- EMX FIFO may-analysis
  - Definite misses
  - Early Miss eXploitation



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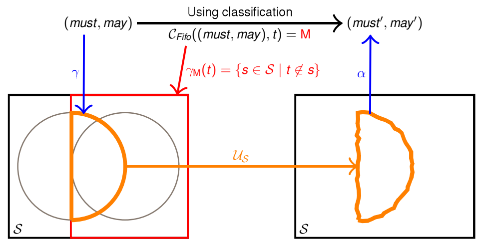

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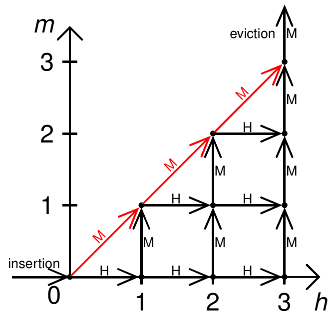


Thank you for listening. Questions?

Further Reading

- R. Wilhelm et al.
  The worst-case execution time problem—
  overview of methods and survey of tools
  Transactions on Embedded Computing Systems, 7(3), 2008
- J. Reineke and D. Grund
  Relative competitive analysis of cache replacement policies
  LCTES 2008

Related Work: LRU Analyses

- Bounds on number of cache misses:

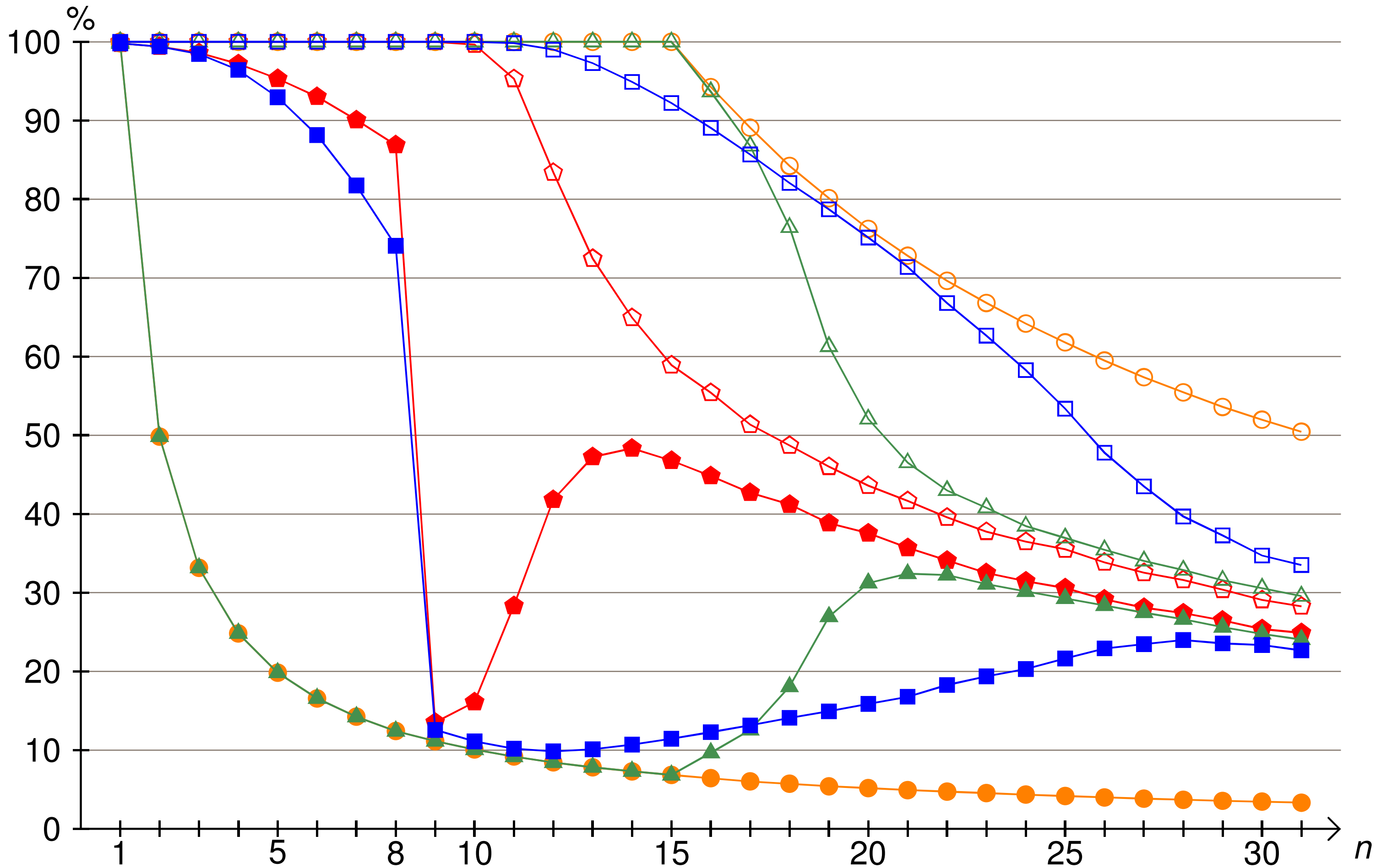
Ghosh	Cache Miss Equations, loop nests
Chatterjee	Exact model of cache behavior for loop nests

- Classification of individual accesses:

Mueller	By "static cache simulation"
Li	By integer linear programming
Ferdinand	By abstract interpretation

- Only for LRU caches

What About The Gap for $n \leq k$?



Does the Initial Cache State Make a Difference?

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- Yes, a FIFO of size k, is (k,k) sensitive

Definition: Sensitivity

Policy P is (f, c) miss-sensitive if

$$m_P(p, s) \leq f \cdot m_P(p', s) + c$$

for all access sequences s and all cache states p, p'.

$\Rightarrow$ For FIFO of size 4, execution time may differ by a factor of 3

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