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Verification of Real-Time Systems
Bounding the Cache-Related Preemption Delay

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Outline

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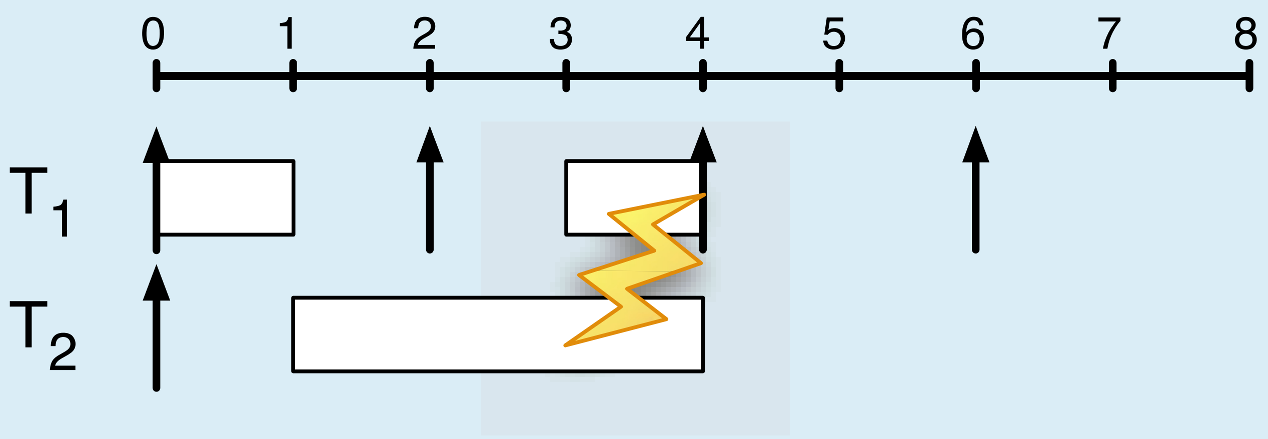
Why use preemptive scheduling?

- Preemption often increases schedulability of task sets.
- Tasks with short deadlines are often unschedulable non-preemptively.

Example

Given: Two periodic tasks T_1 and T_2, with periods $P_1 = 2, P_2 = 8$, deadlines $D_1 = P_1, D_2 = P_2$, and execution times $C_1 = 1, C_2 = 3$.



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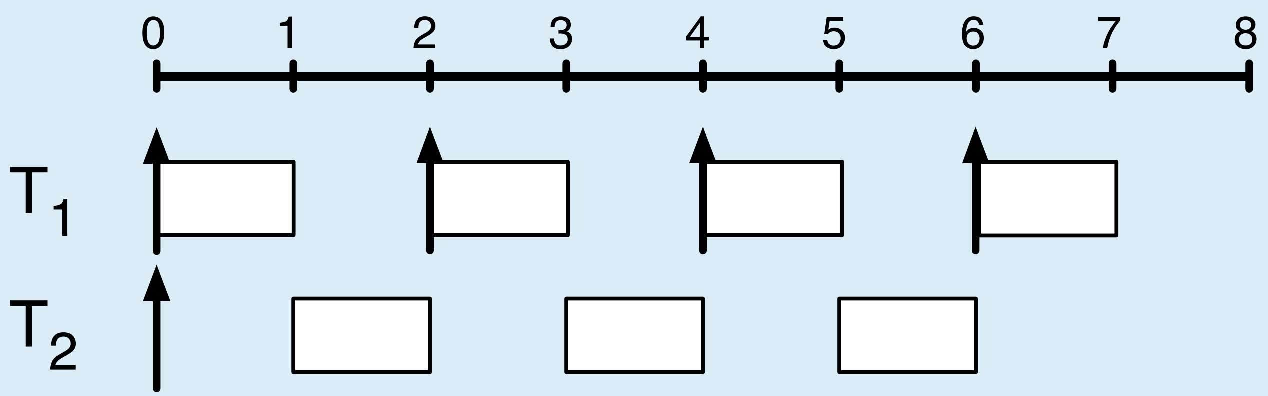
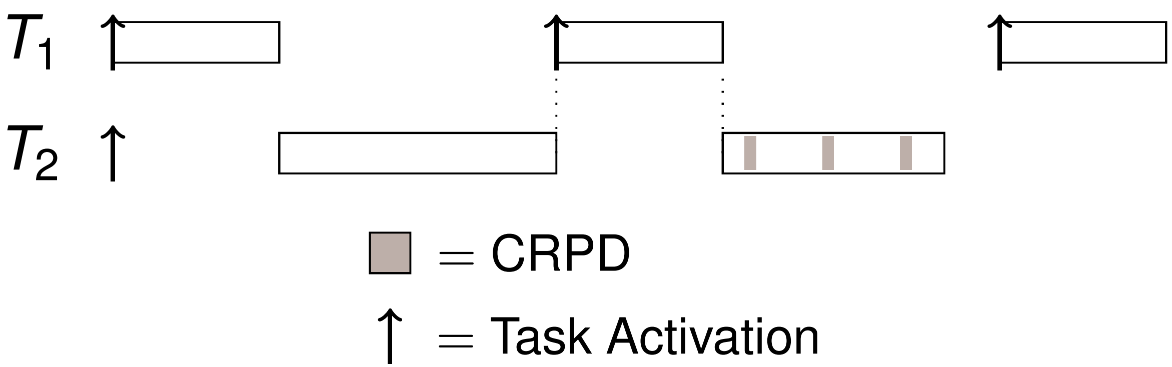


Preemption does not come for free!

- The preempting task “disturbs” the state of performance-enhancing features like caches and pipelines.
- Once the preempted task resumes its execution, the disturbance may cause additional *cache misses*.
- The additional execution time due to additional cache misses is known as the *cache-related preemption delay*.



How to take preemption cost into account?

Where to account for preemption cost?

- Integrate into WCET Analysis: [Schneider, 2000]
  - Assume cache misses everywhere
  - Very pessimistic but easy to use with existing schedulability analyses
- WCET Analysis + CRPD Analysis: [Lee et al., 1996]
  - $WCET_{bound} + n \cdot CRPD_{bound} \geq$ execution time of task with up to n preemptions
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Focus of this lecture: approaches to bound the CRPD
And a bit on: using these bounds within schedulability analyses

Outline

CRPD Analyses

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- Preempted Task:
  How many additional cache misses can a single preemption by *any* preempting task cause in a *given* preempted task?
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- Preempted + Preempting Task
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Analysis of the Preempted Task: Useful Cache Blocks (UCB)

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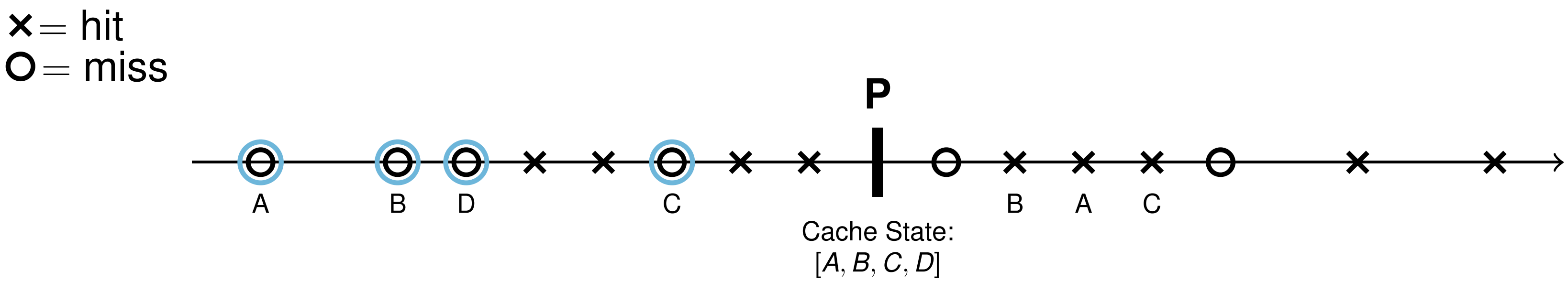
Definition (Useful Cache Block, [Lee et al., 1996])

A memory block *m* at program point *P* is called a useful cache block, if

a) *m* may be cached at *P*

b) *m* may be reused at program point *Q* that may be reached from *P* with no eviction of *m* on this path.



$$\text{UCB} \supseteq \{A, B, C, D\}$$

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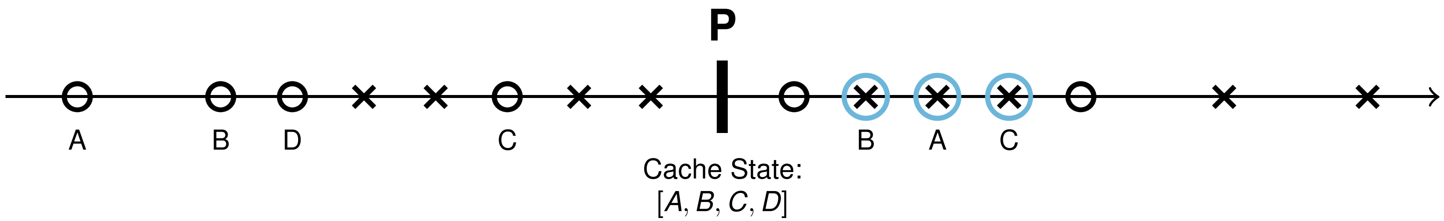
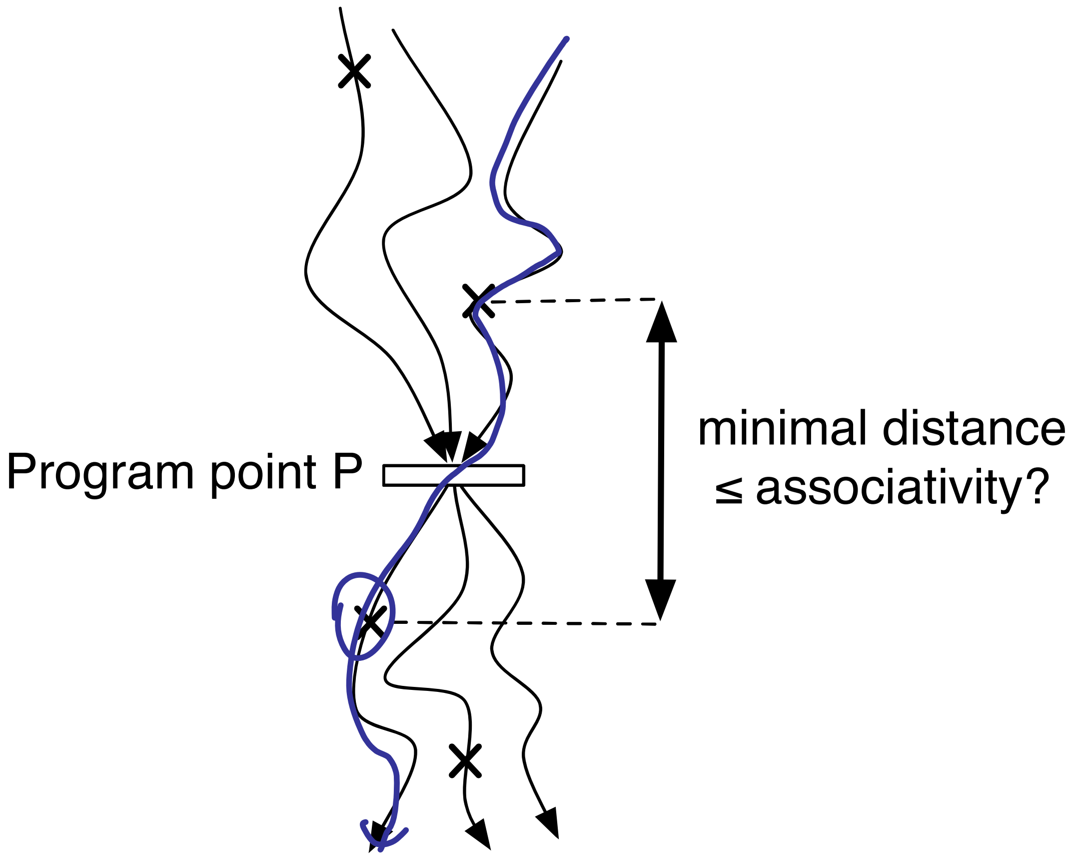
× = hit
O = miss



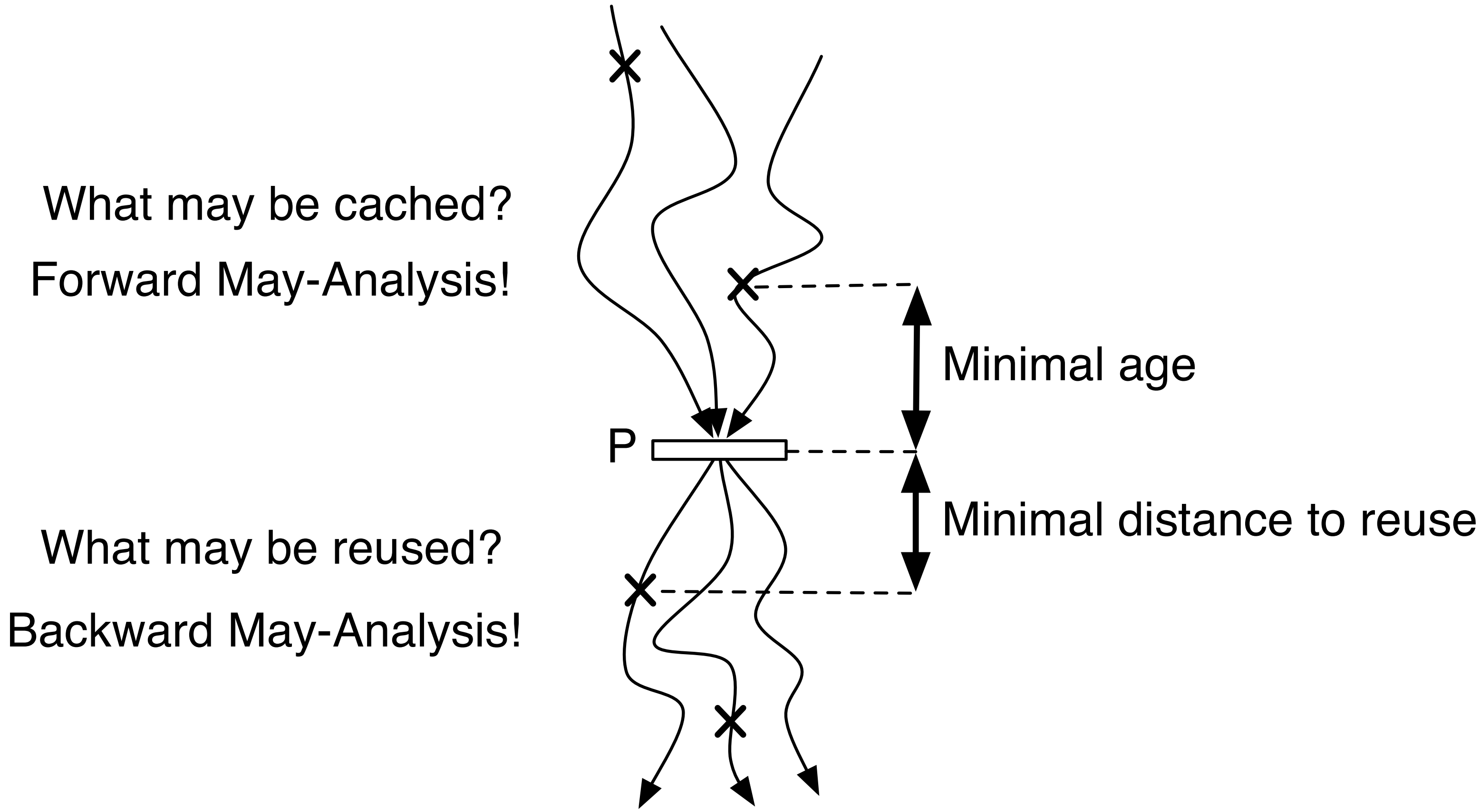
$$\text{UCB} = \{A, B, C\}$$

UCB Analysis



UCB Analysis

Combination of two LRU-may-analyses:

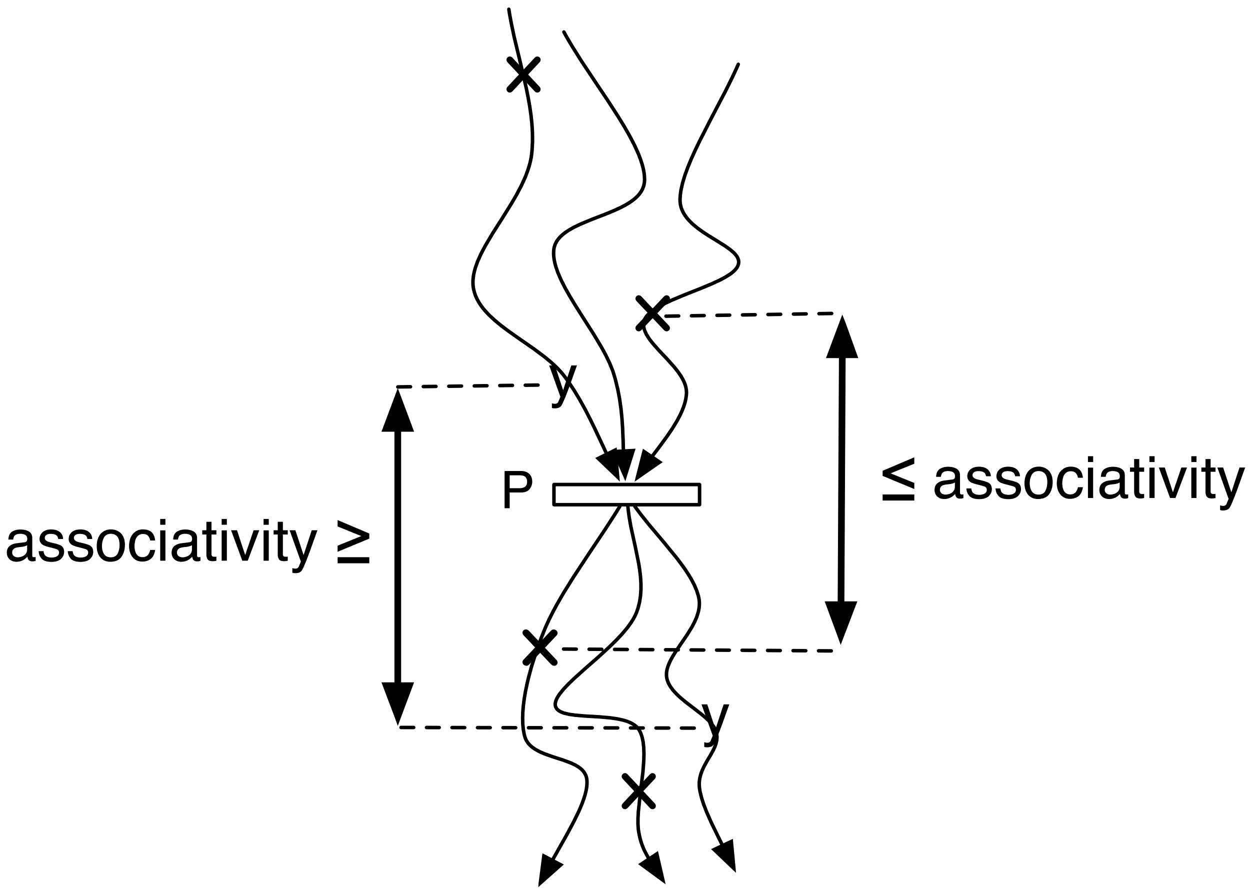


Minimal age + Minimal distance to reuse $\leq$ associativity

$\Longrightarrow$ Memory block may be useful

Improvement: Path Analysis

Some blocks are never useful at the same time:



Literature:

[Tomiyama and Dutt, 2000, Negi et al., 2003, Staschulat and Ernst, 2007]

Improvement: Avoid Accumulating Overestimations

Schedulability analyses rely on:

$$WCET_{bound} + n \cdot CRPD_{bound} \geq \text{exec. time with up to } n \text{ preemptions}$$

n: # of possible preemptions

$CRPD_{bound}$: $BRT \cdot |UCB|$

Yet, we usually have:

$$WCET_{bound} \geq \text{execution time without preemptions}$$
$$CRPD_{bound} \geq \text{additional execution time due to one preemption}$$

$\Longrightarrow$ Overestimation in both analyses adds up:

Some cache misses are counted twice!

Bounding the CRPD using UCBs for Fully-Associative Caches

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- CRPD bound at program point P:

$$\text{CRPD}_{\text{UCB}}^{\text{LRU}}(P) = \text{BRT} \cdot \mathit{min}(|\text{UCB}(P)|, k),$$

where k = associativity and BRT = Block Reload Time

- CRPD bound independent of program point:

$$\text{CRPD}_{\text{UCB}}^{\text{LRU}} = \max_{P} \text{CRPD}_{\text{UCB}}^{\text{LRU}}(P)$$

Slightly more complicated for set-associative caches:
sum up bounds of all cache sets.

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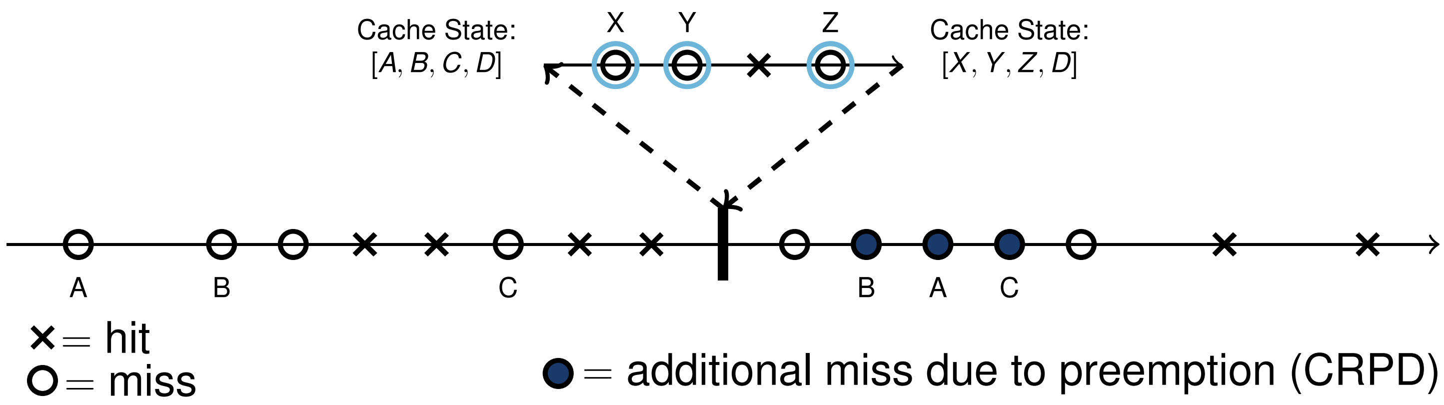
Analysis of the Preempting Task: Evicting Cache Blocks

Definition (Evicting Cache Blocks (ECB), [Tomiyama and Dutt, 2000])

A memory block of the preempting task is called an *evicting cache block*, if it may be accessed during the execution of the preempting task.



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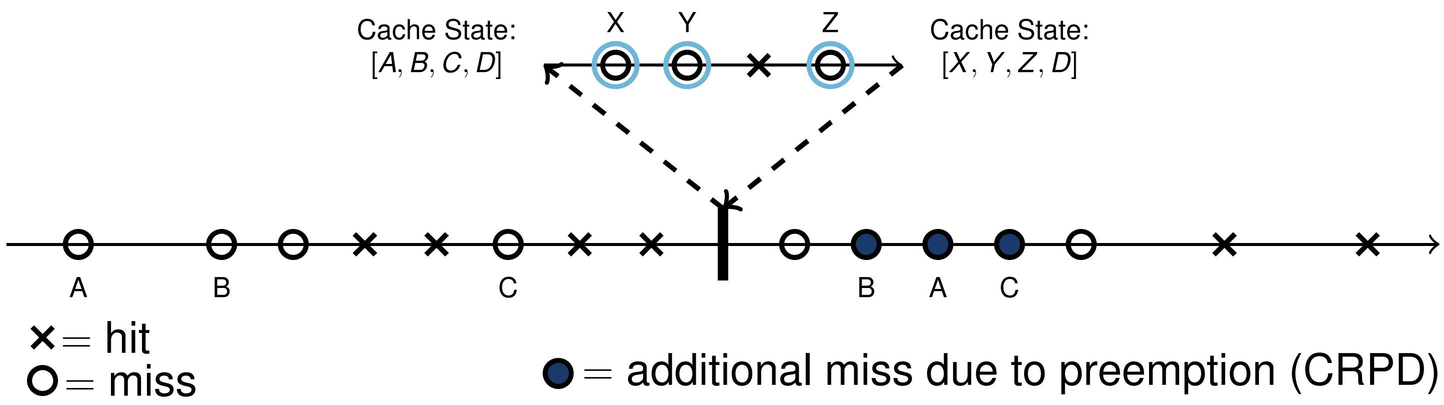
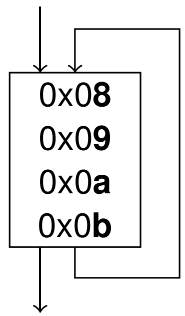


$$\mathrm{CRPD}^{\mathrm{LRU}}_{\mathrm{ECB}} \overset{?}{=} \mathrm{BRT} \cdot min(|\mathrm{ECB}|, k)$$

k = associativity

BRT = Block Reload Time

CRPD Computation for LRU using ECBs: Pitfall



$$[b, a, 9, 8] \xrightarrow{8} [8, b, a, 9] \xrightarrow{9} [9, 8, b, a] \xrightarrow{a} [a, 9, 8, b] \xrightarrow{b} [b, a, 9, 8] \quad \text{0 misses}$$

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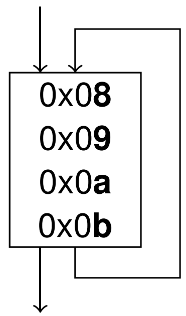

ECBs $= \{\mathbf{e}\}$

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$$[\mathbf{e}, b, a, 9] \xrightarrow{8^*} [8, \mathbf{e}, b, a] \xrightarrow{9^*} [9, 8, \mathbf{e}, b] \xrightarrow{a^*} [a, 9, 8, \mathbf{e}] \xrightarrow{b^*} [b, a, 9, 8] \quad \text{4 misses}$$

- $|\text{UCB}| = 4$
- $|\text{ECB}| = 1$
- $k = \text{associativity} = 4$
- number of additional misses $= 4$

CRPD Computation for LRU using ECB: Sound but Imprecise

- ECB analysis only to determine whether the set is used at all by the preempting task or not:

$$\text{CRPD}^{\text{LRU}}_{\text{ECB}} = \begin{cases} 0 & \text{if ECB} = \emptyset \\ \text{BRT} \cdot k & \text{otherwise} \end{cases}$$

- Cannot do better than that without knowledge of preempted task.

Outline

1 Why Preemptive Scheduling?

2 CRPD Computation
- Analysis of the Preempted Task
- Analysis of the Preempting Task
- Analysis of the Preempted and the Preempting Task
- Simplifying or Eliminating the Problem
- Policies other than LRU

3 Accounting for CRPD within Response-Time Analysis

4 Summary

Analysis of Preempted and Preempting Task: "Shallow" Combination

Take the minimum of the *UCB*- and *ECB*-based estimations.

- CRPD bound for entire program:

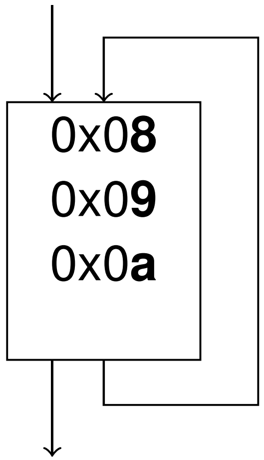
$$\text{CRPD}^{\text{LRU}}_{\text{UCB+ECB}} = \min(\text{CRPD}^{\text{LRU}}_{\text{ECB}}, \text{CRPD}^{\text{LRU}}_{\text{UCB}})$$

Becomes slightly more complicated for set-associative caches:
For a program point: sum of point-wise minima of all cache sets.

Literature:

- For direct-mapped caches: [Negi et al., 2003]
- For set-associative caches:
  [Tan and Mooney, 2004, Burguière et al., 2009]

Analysis of Preempted and Preempting Task: "Deeper" Combination



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Without preemption:

$$[a, 9, 8, 7] \xrightarrow{8} [8, a, 9, 7] \xrightarrow{9} [9, 8, a, 7] \xrightarrow{a} [a, 9, 8, 7]$$

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Some of the UCBs are guaranteed to *remain useful* under preemption!

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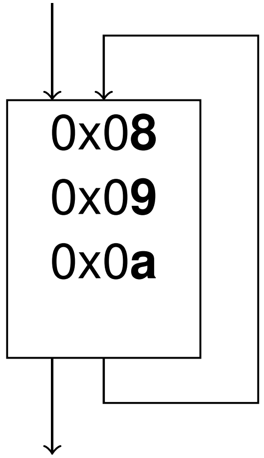

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- $\text{CRPD}_{\text{UCB\&ECB}} = \min(\text{CRPD}_{\text{UCB}}, \text{CRPD}_{\text{ECB}}) = \min(3, 4) = 3$
- Yet: actual number of additional misses: 0

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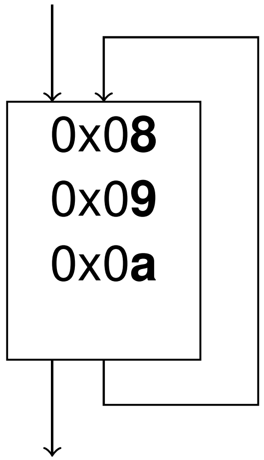

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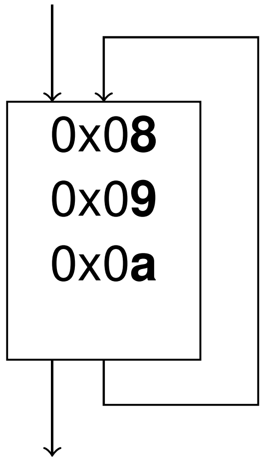

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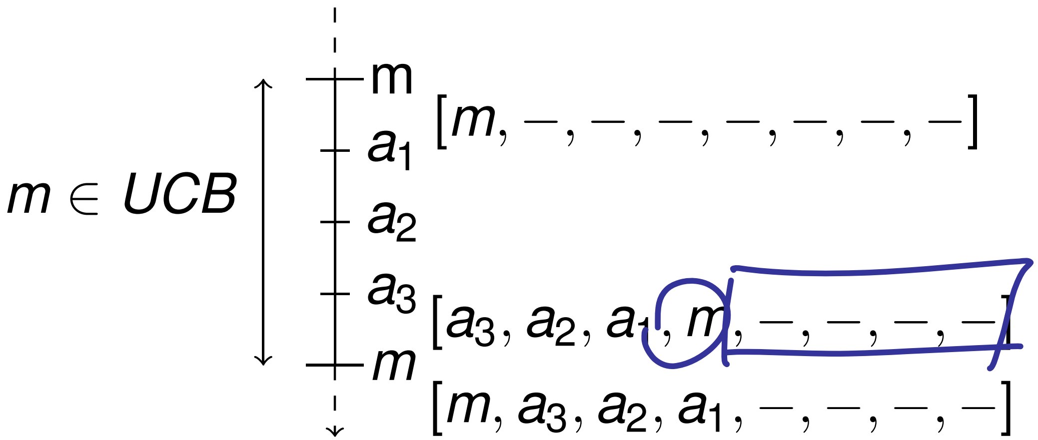
Why?

- Minimal number of ECBs to evict a UCB is 2, but $|\text{ECB}| = 1$
- A single ECB is not sufficient to evict any of the UCBs.

Combining UCB and ECB: Notion of Resilience

Determining the maximal number of ECBs, such that no additional cache miss may occur:



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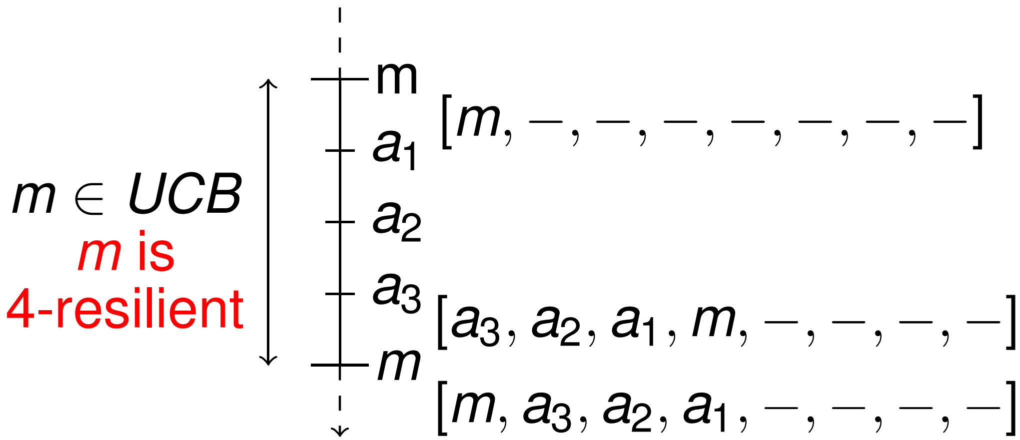

Resilience Analysis

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Definition (Resilience)

A memory block m is called *l-resilient* at program point P, if all possible next accesses to m

- that would be hits without preemption,
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- No UCB is k-resilient, i.e., no UCB remains useful after a preemption with k (= associativity) many ECBs.
- Each $(l + 1)$-resilient UCB is also l-resilient.
- Each UCB is at least 0-resilient.

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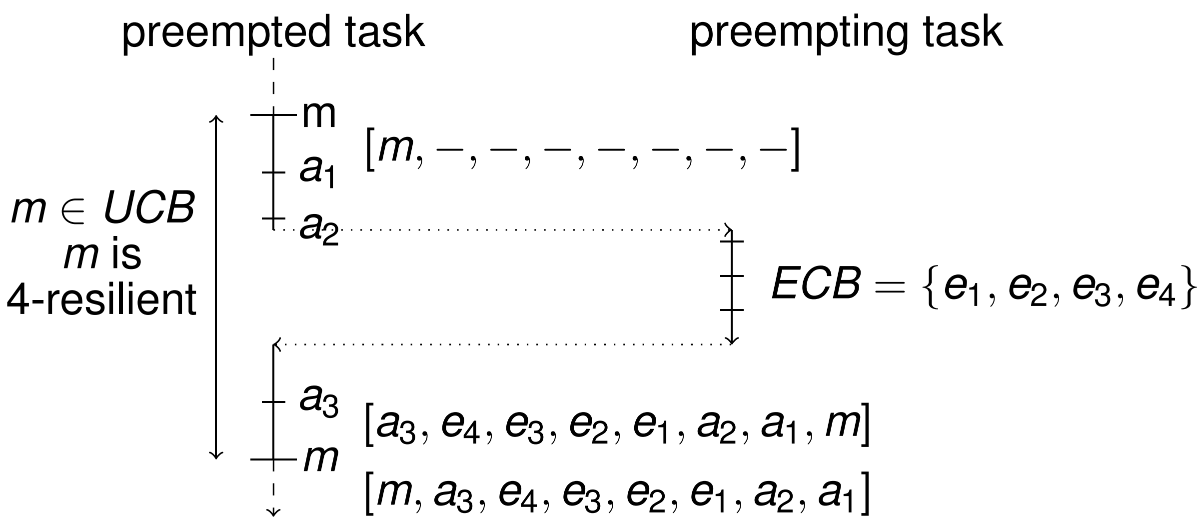

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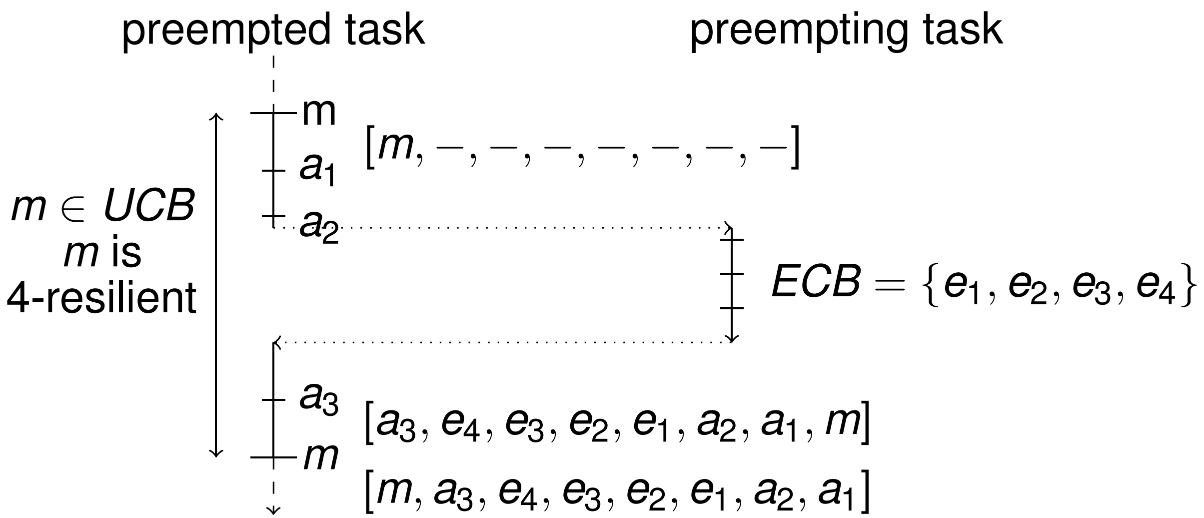

In general: if $|\text{ECB}| \leq l$ then the UCB is not evicted

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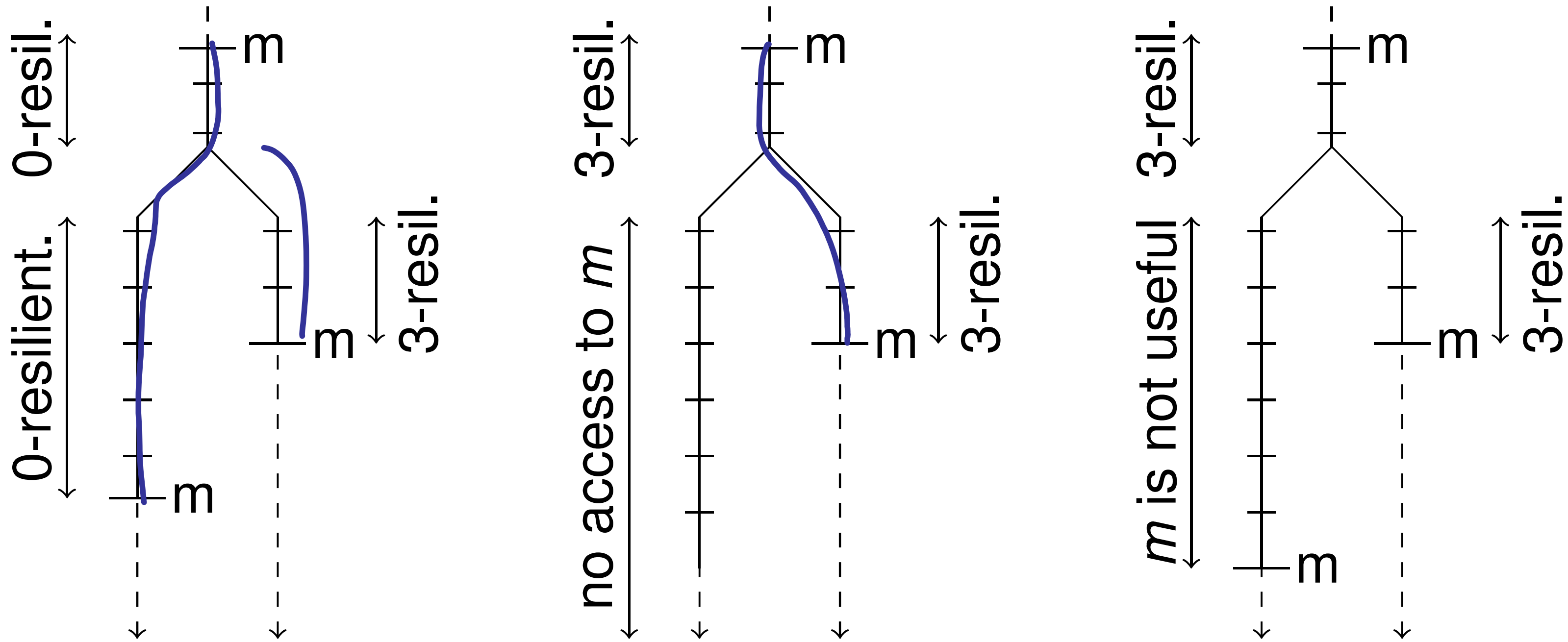

Bounding the CRPD using Resilience

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CRPD (Combining UCB and ECB by using Resilience)

$$\left|\overbrace{\underbrace{UCB}_{\textit{useful}} \setminus \underbrace{\{m \mid m \textit{ is } |ECB|\textit{-resilient}\}}_{\textit{remain useful}}}^{\textit{blocks contributing to CRPD}}\right|$$

Bounding the CRPD using Resilience

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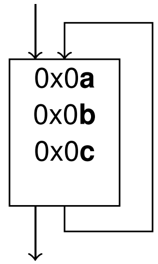
CRPD (Combining UCB and ECB by using Resilience)

$$CRPD \leq BRT \times |\overbrace{\underbrace{UCB}_{useful} \setminus \underbrace{\{m \mid m \text{ is } |ECB|\text{-resilient}\}}_{remain\ useful}}^{blocks\ contributing\ to\ CRPD}|$$

Bounding the CRPD using Resilience: Example



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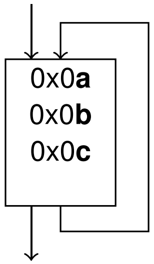

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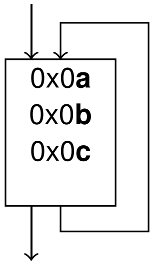
- $|\text{ECB}| = 1$
- a, b, and c are 1-resilient
- $CRPD^{res}_{\text{UCB\&ECB}} = BRT \times |UCB \setminus \{m \mid m \text{ is } |\text{ECB}|\text{-resilient}\}| = 0$

= UCB

Bounding the CRPD using Resilience: Example



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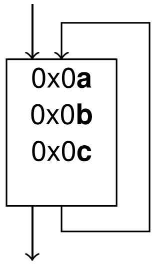
Bounding the CRPD using Resilience: Example



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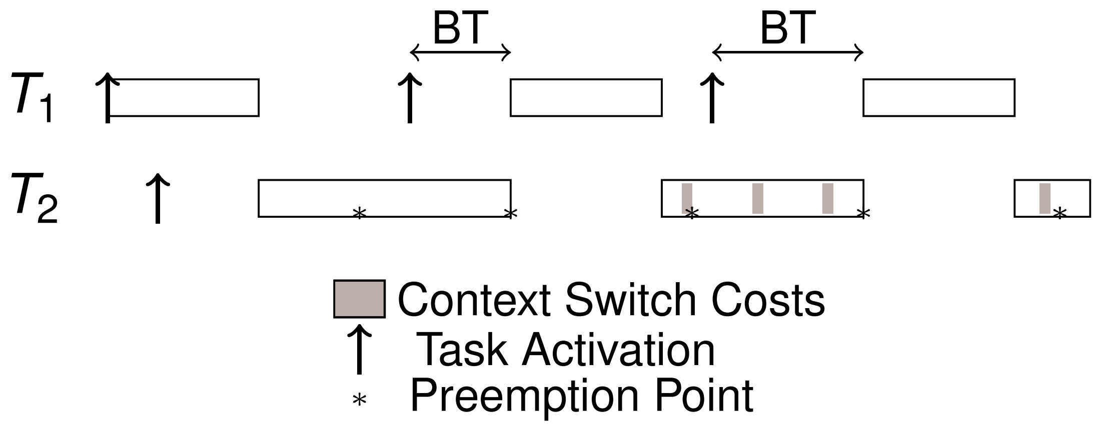
Outline

Deferred Preemption

- Restrict preemptions to a set of predefined *preemption points*.
- Introduces new problem: blocking time, i.e., time until next preemption point is reached.
  Intervals between preemption points $\equiv$ critical sections.



Where to place preemptions points, s.t.

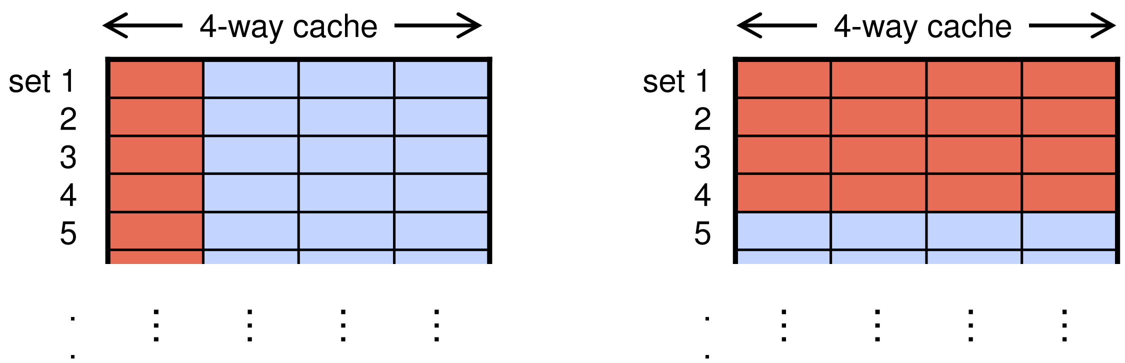
- CRPD is minimized, and
- *Maximum Blocking Time* is minimized?

Analysis to determine maximum blocking time for given set of preemption points: [Lee et al., 1998, Altmeyer et al., 2009]

Cache Partitioning

Additional cache misses are due to interference on the cache.

$\Longrightarrow$ Cache Partitioning eliminates this interference.



- Software-based Cache Partitioning [Wolfe, 1994, Mueller, 1995]:
  - Change layout of instructions and data such that tasks map to disjoint cache sets
  - Particularly difficult for large arrays
- Hardware-based Cache Partitioning [Kirk and Strosnider, 1990, Chiou, 1999]:
  - Partition cache by cache sets and/or cache ways
  - Increases hardware cost
  - Renewed interest in multi-cores with shared caches

Outline

Do existing approaches work for FIFO, PLRU, etc.?

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Plain answer: No!

Do existing approaches work for FIFO, PLRU, etc.?

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Plain answer: No!

Counterexample for FIFO [Burguière et al., 2009]:

$$\text{ECBs} = \{\mathbf{x}\} \begin{cases} [b,a] \xrightarrow{a} [b,a] \xrightarrow{e^*} [e,b] \xrightarrow{b} [e,b] \xrightarrow{c^*} [c,e] \xrightarrow{e} [c,e] & \text{2 misses} \\ [\mathbf{x},b] \xrightarrow{a^*} [a,\mathbf{x}] \xrightarrow{e^*} [e,a] \xrightarrow{b^*} [b,e] \xrightarrow{c^*} [c,b] \xrightarrow{e^*} [e,c] & \text{5 misses} \end{cases}$$

- $|\mathrm{UCB}(s)| = 2$
- $|\mathrm{ECB}(s)| = 1$
- associativity $k = 2$
- But: number of additional misses$= 3$

Same result for PLRU.

Idea [Burguière et al., 2009]: Use Relative Competitiveness Results

Some relative competitiveness results:

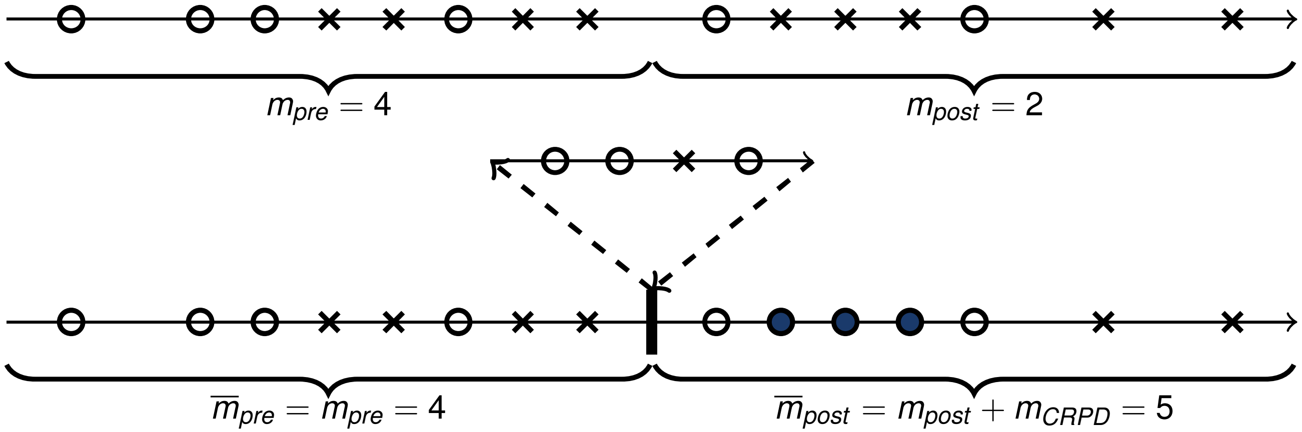
- PLRU(n) is $(1, 0)$-miss-competitive relative to LRU($1 + log_2 n$).
- FIFO(n) is $(\frac{n}{n-r+1}, r)$-miss-competitive relative to LRU(r).

$\Longrightarrow$ Performing WCET and CRPD analyses assuming LRU($1 + log_2 n$) replacement should give correct bounds for PLRU(n).

Can we also make use of non-$(1, 0)$-competitiveness?

Applying Relative Competitiveness: A sequence of memory accesses

- Notation:
  - m = number of misses
  - $\overline{m}$ = number of misses in the case of preemption



Applying Relative Competitiveness: A sequence of memory accesses

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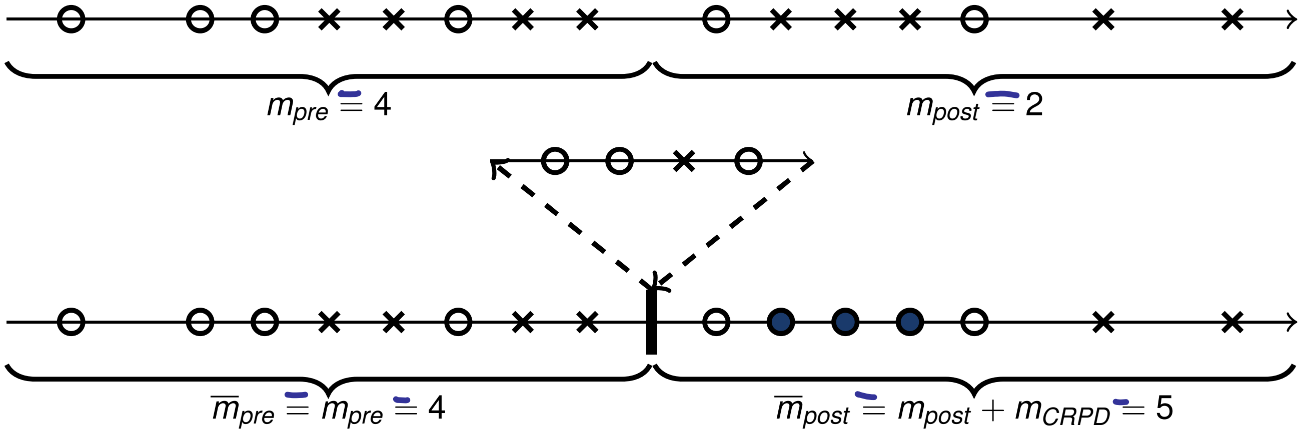

- Assume P(t) is (k, c)-miss-competitive rel. to LRU(s). Then:

$$\overline{m}^{P(t)} = \overline{m}^{P(t)}_{pre} + \overline{m}^{P(t)}_{post}$$

Applying Relative Competitiveness: A sequence of memory accesses

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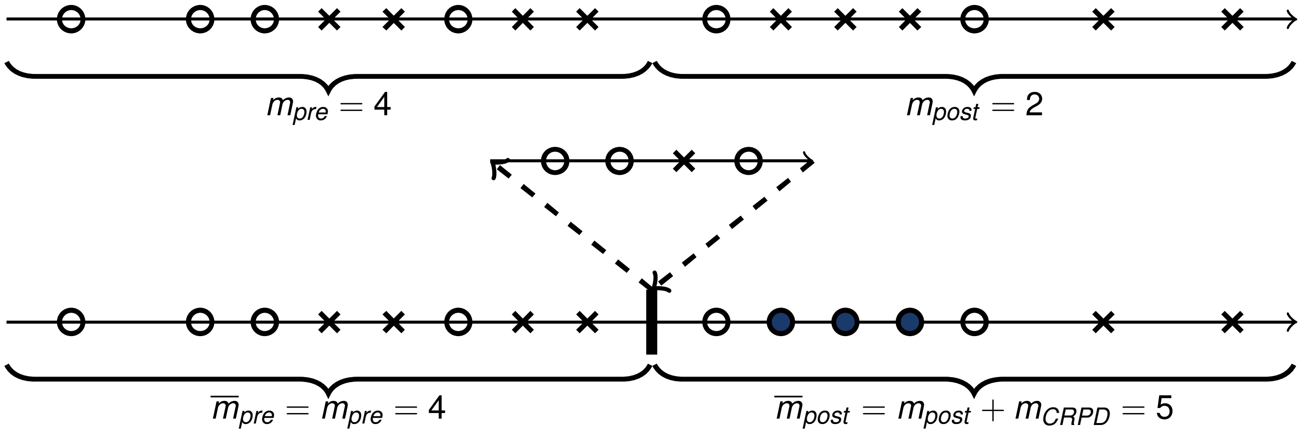

- Assume $\mathrm{P}(t)$ is (k, c)-miss-competitive rel. to $\mathrm{LRU}(s)$. Then:

$$\begin{aligned}\overline{m}^{\mathrm{P}(t)} &= \overline{m}_{pre}^{\mathrm{P}(t)} + \overline{m}_{post}^{\mathrm{P}(t)} \\ &\leq [k \cdot m_{pre}^{\mathrm{LRU}(s)} + c] + [k \cdot (m_{post}^{\mathrm{LRU}(s)} + m_{CRPD}^{\mathrm{LRU}(s)}) + c]\end{aligned}$$

Applying Relative Competitiveness: A sequence of memory accesses

- Notation:
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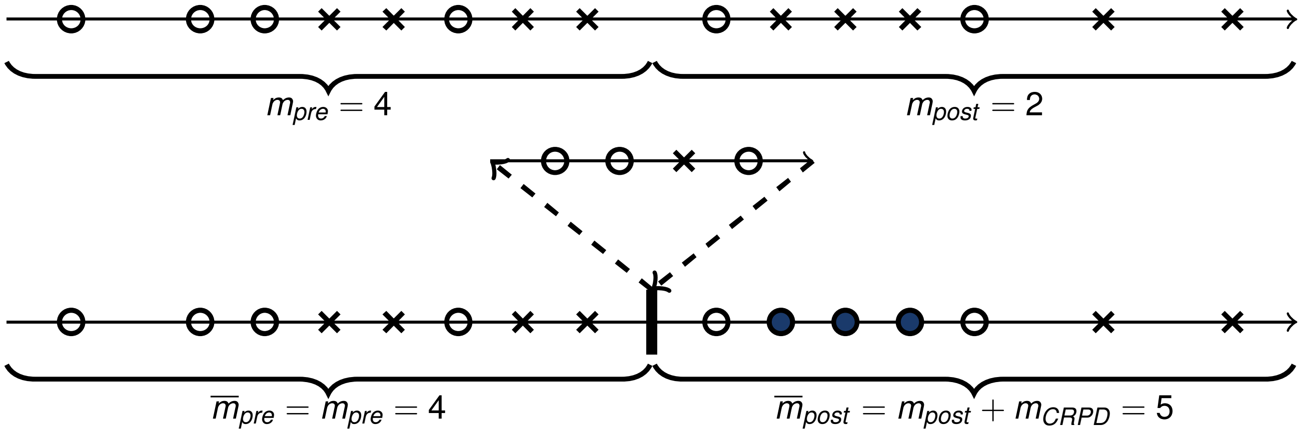

- Assume P(t) is (k, c)-miss-competitive rel. to LRU(s). Then:

$$\begin{aligned}
\overline{m}^{P(t)} &= \overline{m}_{pre}^{P(t)} + \overline{m}_{post}^{P(t)} \\
&\leq [k \cdot m_{pre}^{LRU(s)} + c] + [k \cdot (m_{post}^{LRU(s)} + m_{CRPD}^{LRU(s)}) + c] \\
&= [k \cdot m^{LRU(s)} + c] + [k \cdot m_{CRPD}^{LRU(s)} + c]
\end{aligned}$$

Applying Relative Competitiveness

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- Assume $\mathrm{P}(t)$ is (k, c)-miss-competitive rel. to $\mathrm{LRU}(s)$. Then:

$$\overline{m}^{\mathrm{P}(t)} \leq [k \cdot m^{\mathrm{LRU}(s)} + c] + [k \cdot m_{CRPD}^{\mathrm{LRU}(s)} + c]$$

- In WCET analysis:
  Take into account $k \cdot m^{\mathrm{LRU}(s)} + c$ misses
- In CRPD analysis:
  Take into account $k \cdot m_{CRPD}^{\mathrm{LRU}(s)} + c$ misses

Outline

Recap: Rate-Monotonic Schedulability Condition

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The time-demand function $W_i(t)$ of the task τ_i is defined as follows:

$$W_i(t) = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{t}{T_j} \right\rceil C_j.$$

Theorem

A system $\mathcal{T}$ of periodic, independent, preemptable tasks is schedulable on one processor by algorithm A if it holds that:

$$\forall \tau_i \in \mathcal{T}\ \exists t \text{ with } 0 < t \leq D_i \text{ and } W_i(t) \leq t$$

This condition is also necessary for synchronous, periodic task sets and also sporadic task sets.

Note that this holds for implicit-deadline and constrained-deadline task sets.

Recap: Response-Time Analysis for RM

Want to determine whether there is a t such that $W_i(t) \leq t \leq D_i$.

Compute smallest such t (if there is any) by fixed-point iteration:

$n := 0$
$R_i^0 := C_i$
do
$n := n + 1$
$$R_i^n := C_i + \sum_{j=1}^{i-1} \left\lceil \frac{R_i^{n-1}}{T_j} \right\rceil C_j$$
while $R_i^n < D_i$ and $R_i^{n-1} < R_i^n$

A set of tasks is schedulable if $R_i^n < D_i$ for every task i.

How to incorporate CRPD?

Introduce additional term $\gamma_{i,j}$ in the time-demand function:

$$W_i(t) = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{t}{T_j} \right\rceil (C_j + \gamma_{i,j}).$$

Fixed-point iteration can be appropriately adapted.

How to incorporate CRPD?

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Fixed-point iteration can be appropriately adapted.

What is the meaning of $\gamma_{i,j}$?

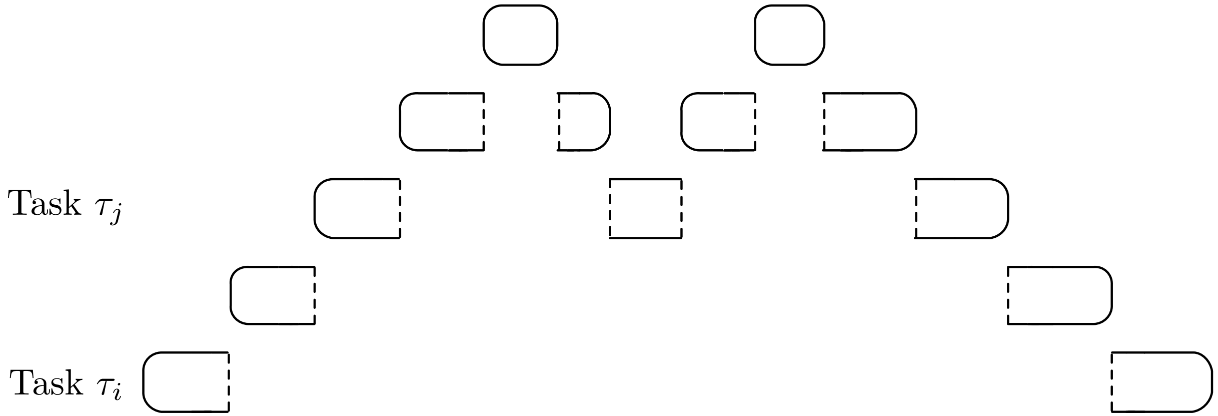
Which value should it take?

Interpretation of $\gamma_{i,j}$

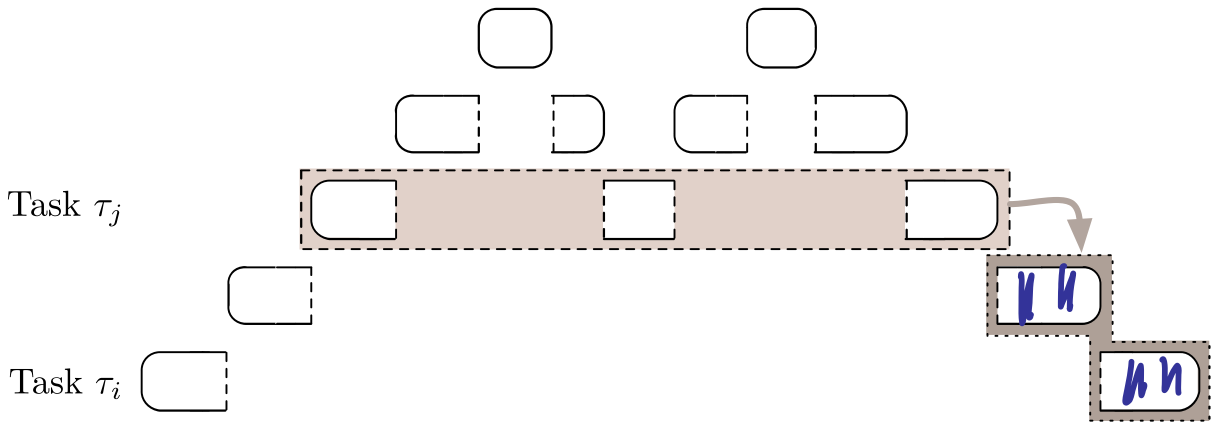
Without nested preemptions:

$$\gamma_{i,j} = \text{CRPD due to one preemption of task } i \text{ by task } j.$$

But what if there are nested preemptions?

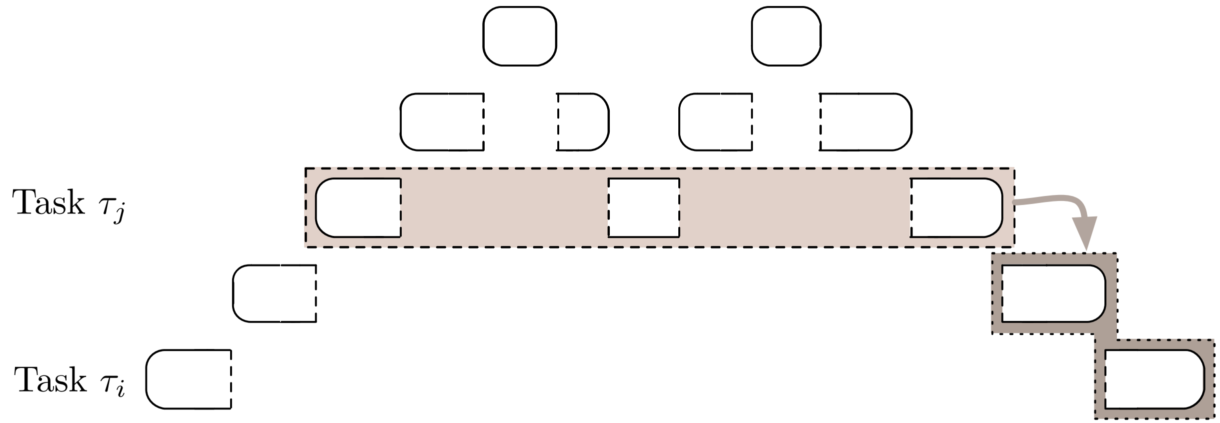


First Interpretation of $\gamma_{i,j}$: "Effect of preempting task"



Under this interpretation, $\gamma_{i,j}$ needs to bound the effect of *j*'s execution on all tasks of lower priority up to task *i*.

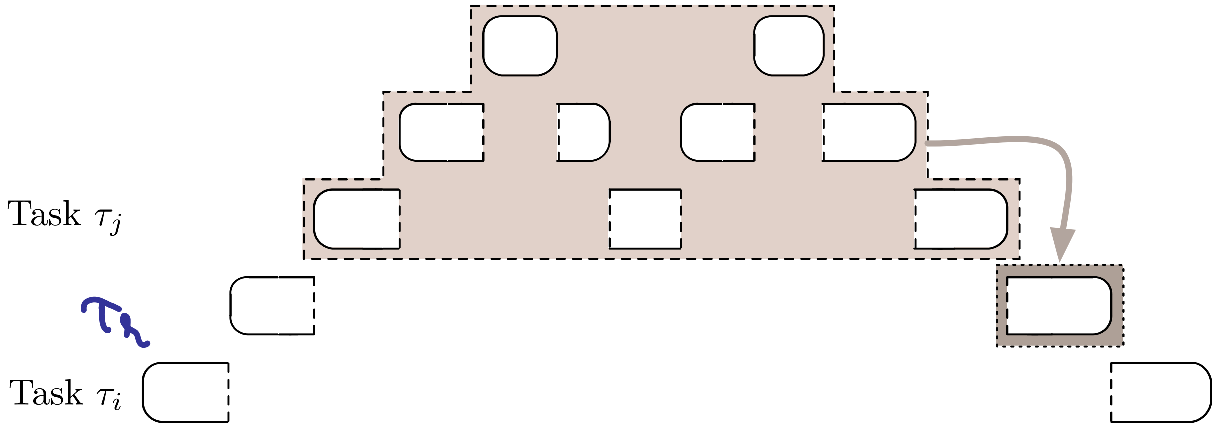
First Interpretation of $\gamma_{i,j}$: "Effect of preempting task"



For *direct-mapped caches* there are three such approaches:

1. "ECB-Only": $\gamma_{i,j} = BRT \cdot |ECB_j|$
2. "UCB-Union": $\gamma_{i,j} = BRT \cdot \left|\bigcup_{k \in aff(i,j)} UCB_k\right|$, where $aff(i,j) = hep(i) \cap lp(j)$.
3. Combination of the above: $\gamma_{i,j} = BRT \cdot \left|\bigcup_{k \in aff(i,j)} UCB_k \cap ECB_j\right|$

Second Interpretation of $\gamma_{i,j}$: "Effect of immediately preempted task"



Under this interpretation, $\gamma_{i,j}$ needs to bound the effect of *j*'s execution and that of the execution of task's preempting *j* itself on the task that *j* immediately preempted.

Second Interpretation of $\gamma_{i,j}$: "Effect of immediately preempted task"

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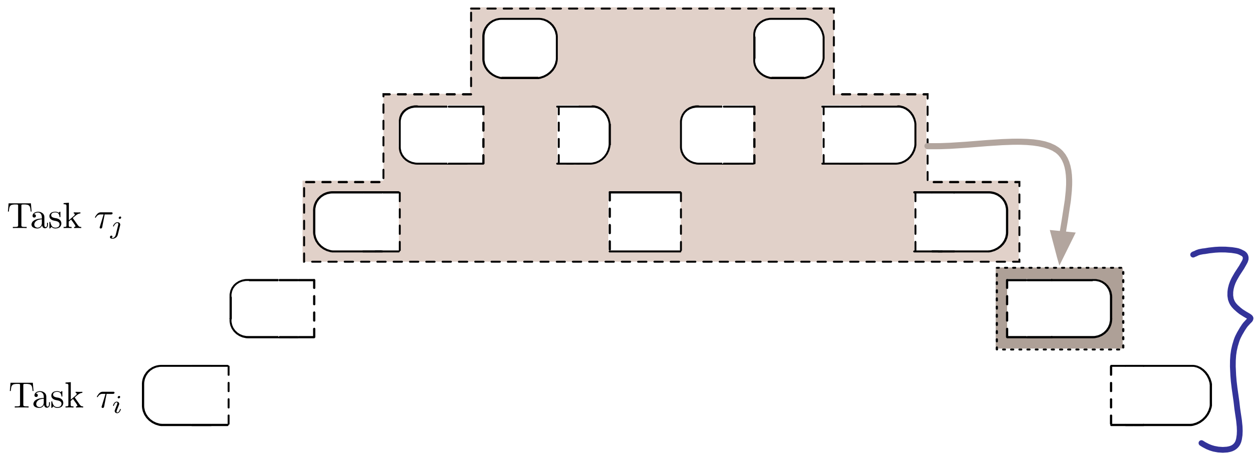
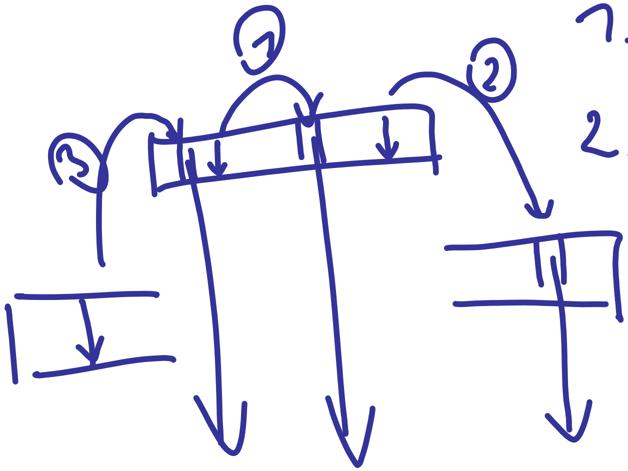


For *direct-mapped caches* there are three such approaches:

1. "UCB-Only": $\gamma_{i,j} = BRT \cdot \max_{k \in aff(i,j)} |UCB_k|$
2. "ECB-Union": $\gamma_{i,j} = BRT \cdot \left|\bigcup_{h \in hp(j) \cup \{j\}} ECB_h\right|$.
3. Combination of the above:
$\gamma_{i,j} = BRT \cdot \max_{k \in aff(i,j)} \left|UCB_k \cap \bigcup_{h \in hp(j) \cup \{j\}} ECB_h\right|$.

Some Open Problems

- Soundly and precisely handle set-associative caches in response-time analysis and use *resilience* information.
- Empirical analysis of sources of imprecision: is overestimation due to characterization of preempting tasks, of preempted task, or other imprecisions?
- WRITE-BACK CACHES



1. TASK-INTERNAL WBS
2. PREEMPTION-INDUCED WBs
3. CARRY-IN WBS

Outline

Summary

- Preemptive Scheduling desirable, but not for free:
  $\longrightarrow$ Need to bound CRPD
- For LRU, the CRPD can be bounded by analyzing
  - the preempted task: UCB analysis
  - the preempting task: ECB analysis
    - Sound approach rather imprecise
    - Need to couple more tightly with analysis of preempted task
  - both, the preempted and the preempting task
    - "Shallow" combination
    - "Deeper" combination: Resilience analysis
- Approaches do not carry over to FIFO, PLRU, etc. immediately
  - First approach: relative competitiveness
- Can be accounted for in response-time analysis in different ways
  - Has so far only been investigated in detail for direct-mapped caches

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