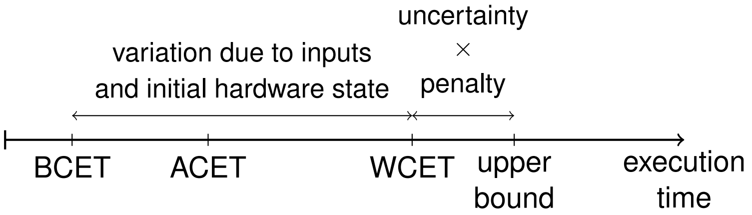
Outline

1. Caches
2. Cache Analysis for Least-Recently-Used
3. Beyond Least-Recently-Used
   - Predictability Metrics
   - Relative Competitiveness
   - Sensitivity – Caches and Measurement-Based Timing Analysis
4. Summary

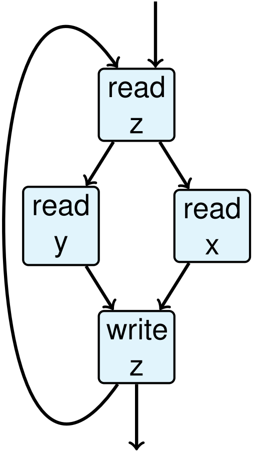
Outline

Uncertainty in WCET Analysis

- Amount of uncertainty determines precision of WCET analysis
- Uncertainty in cache analysis depends on replacement policy



Uncertainty in Cache Analysis



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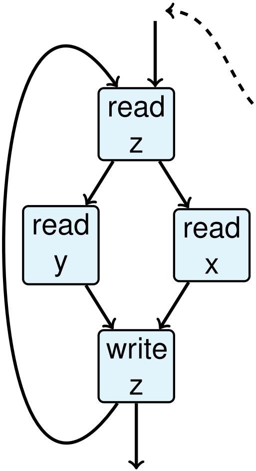

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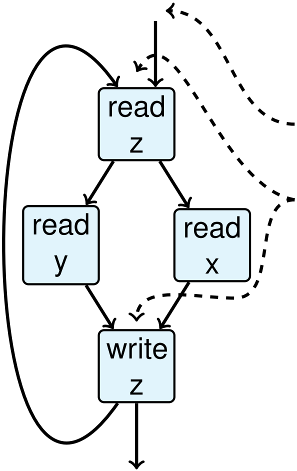
Uncertainty in Cache Analysis



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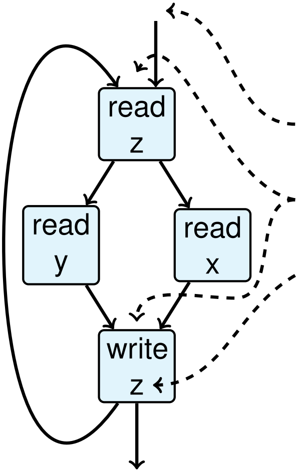

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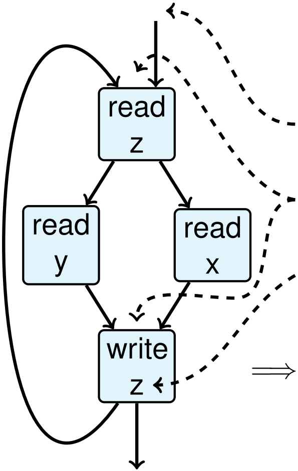
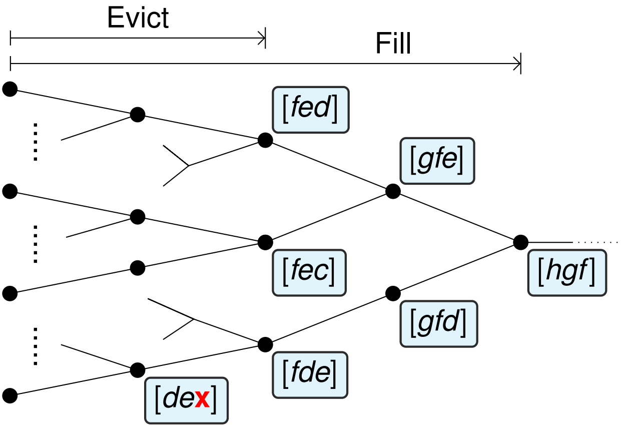


1. Initial cache contents unknown.
2. Need to combine information.
3. Cannot resolve address of *z*.

$\Longrightarrow$ Amount of uncertainty determined by ability to recover information

Predictability Metrics



Sequence: ⟨a, . . . , e, f, g, h⟩

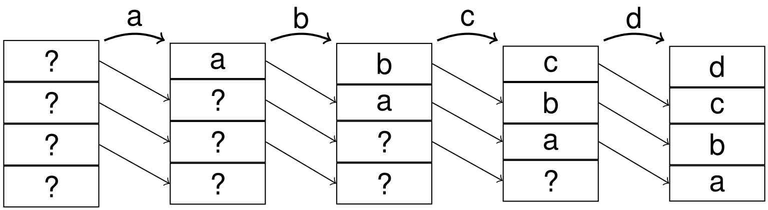
Meaning of Metrics

- Evict
  - Number of accesses to obtain *any may*-information.
  - I.e. when can an analysis predict any cache misses?
- Fill
  - Number of accesses to complete *may*- and *must*-information.
  - I.e. when can an analysis predict each access?

$\longrightarrow$ Evict and Fill bound the precision of *any* static cache analysis.
Can thus serve as a benchmark for analyses.

Evaluation of Least-Recently-Used

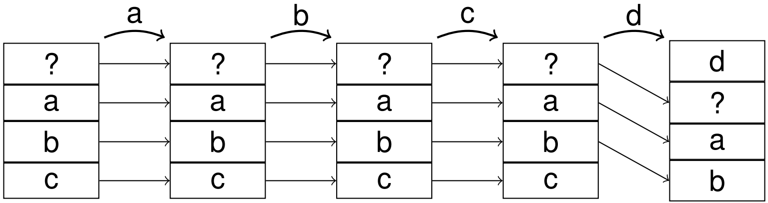
- LRU "forgets" about past quickly:
  - cares about most-recent access to each block only
  - order of previous accesses irrelevant



- In the example: Evict $=$ Fill $= 4$
- In general: Evict$(k) =$ Fill$(k) = k$, where k is the associativity of the cache

Evaluation of First-In First-Out (sketch)

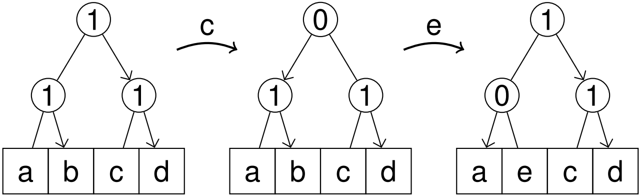
- Like LRU in the miss-case
- But: “Ignores” hits



- In the worst-case $k-1$ hits and k misses: ($k =$ associativity)
  $\longrightarrow$ $\text{Evict}(k) = 2k-1$
- Another k accesses to obtain complete knowledge:
  $\longrightarrow$ $\text{Fill}(k) = 3k-1$

Evaluation of Pseudo-LRU (sketch)

- Tree-bits point to block to be replaced



- Accesses "rejuvenate" neighborhood
  - Active blocks keep their (inactive) neighborhood in the cache
- Analysis yields:
  - $\text{Evict}(k) = \frac{k}{2} \log_2 k + 1$
  - $\text{Fill}(k) = \frac{k}{2} \log_2 k + k - 1$

Evaluation of Policies

Policy	Evict(k)	Fill(k)	Evict(8)	Fill(8)
LRU	k	k	8	8
FIFO	$2k-1$	$3k-1$	15	23
MRU	$2k-2$	$\infty/3k-4$	14	$\infty/20$
PLRU	$\frac{k}{2}\log_2 k+1$	$\frac{k}{2}\log_2 k+k-1$	13	19

- LRU is optimal w.r.t. metrics.
- Other policies are much less predictable.

$\longrightarrow$ Use LRU if predictability is a concern.

- How to obtain *may*- and *must*-information within the given limits for other policies?

Outline

Relative Competitiveness

- Competitiveness (Sleator and Tarjan, 1985):
  worst-case performance of an online policy *relative to the optimal offline policy*
  - used to evaluate online policies

- Relative competitiveness (Reineke and Grund, 2008):
  worst-case performance of an online policy *relative to another online policy*
  - used to derive local and global cache analyses

Definition – Relative Miss-Competitiveness

Notation

$$m_{\mathbf{P}}(p, s) \quad = \quad \textit{number of misses that policy } \mathbf{P} \textit{ incurs on access sequence } s \in M^* \textit{ starting in state } p \in C^{\mathbf{P}}$$

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Definition (Relative miss competitiveness)

Policy **P** is (k, c)-miss-competitive relative to policy **Q** if

$$m_{\mathbf{P}}(p, s) \leq k \cdot m_{\mathbf{Q}}(q, s) + c$$

for all access sequences $s \in M^*$ and cache-set states $p \in C^{\mathbf{P}}, q \in C^{\mathbf{Q}}$ that are compatible $p \sim q$.

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Definition (Competitive miss ratio of **P** relative to **Q**)

The smallest k, s.t. **P** is (k, c)-miss-competitive rel. to **Q** for some c.

Example – Relative Miss-Competitiveness

P is $(3, 4)$-miss-competitive relative to **Q**.
If **Q** incurs x misses, then **P** incurs at most $3 \cdot x + 4$ misses.

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Worst: **P** is not-miss-competitive (or ∞-miss-competitive) relative to **Q**.

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If **Q** has x hits, then **P** has at least $\frac{2}{3} \cdot x - 3$ hits.

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Equivalent to $(1, 0)$-miss-competitiveness.

Worst: **P** is $(0, 0)$-hit-competitive relative to **Q**.
Analogue to ∞-miss-competitiveness.

Local Guarantees: $(1, 0)$-Competitiveness

Let **P** be $(1, 0)$-competitive relative to **Q**:

$$m_{\mathbf{P}}(p, s) \leq 1 \cdot m_{\mathbf{Q}}(q, s) + 0$$
$$\Leftrightarrow m_{\mathbf{P}}(p, s) \leq m_{\mathbf{Q}}(q, s)$$

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1. If **Q** "hits", so does **P**, and
2. if **P** "misses", so does **Q**.

As a consequence,

1. a *must*-analysis for **Q** is also a *must*-analysis for **P**, and
2. a *may*-analysis for **P** is also a *may*-analysis for **Q**.

Global Guarantees: (k, c)-Competitiveness

Given: Global guarantees for policy **Q**.
Wanted: Global guarantees for policy **P**.

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1. Determine competitiveness of policy **P** relative to policy **Q**.

$$\mathbf{m_P} \leq \mathbf{k} \cdot \mathbf{m_Q} + \mathbf{c}$$

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2. Compute global guarantee for task T under policy **Q**.

$$\mathbf{m_Q(T)}$$

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   $\mathbf{m_P \leq k \cdot m_Q + c}$

2. Compute global guarantee for task T under policy **Q**.

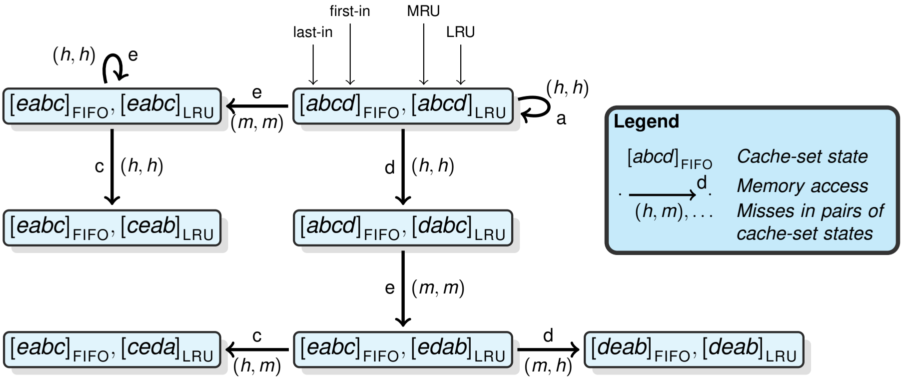
   $\mathbf{m_Q(T)}$

3. Calculate global guarantee on the number of misses for **P** using the global guarantee for **Q** and the competitiveness results of **P** relative to **Q**.

   $\mathbf{m_P \leq k \cdot m_Q + c} \quad \mathbf{m_Q(T)} \quad = \quad \mathbf{m_P(T)}$

Relative Competitiveness: Automatic Computation

P and **Q** (here: FIFO and LRU) induce transition system:



Competitive miss ratio = maximum ratio of misses in policy **P** to misses in policy **Q** in transition system

Transition System is ∞ Large

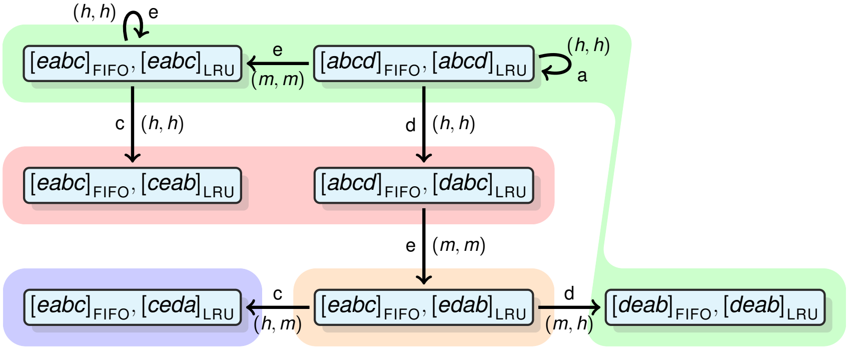
Problem: The induced transition system is ∞ large.

Observation: Only the *relative positions* of elements matter:

$$
\begin{tikzcd}
\boxed{[abc]_{\mathrm{LRU}}, [bde]_{\mathrm{FIFO}}} \arrow[d, "c"', "{(h, m)}"] \arrow[r, phantom, "\approx"] & \boxed{[fgl]_{\mathrm{LRU}}, [ghm]_{\mathrm{FIFO}}} \arrow[d, "l"', "{(h, m)}"] \\
\boxed{[cab]_{\mathrm{LRU}}, [cbd]_{\mathrm{FIFO}}} \arrow[r, phantom, "\approx"] & \boxed{[lfg]_{\mathrm{LRU}}, [lgh]_{\mathrm{FIFO}}}
\end{tikzcd}
$$

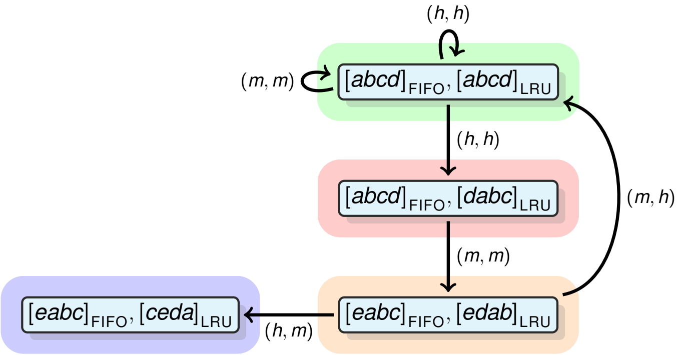
Solution: Construct *finite* quotient transition system.

≈-Equivalent States in Running Example



Finite Quotient Transition System

Merging $\approx$-equivalent states yields a finite quotient transition system:



Competitive Ratio = Maximum Cycle Ratio

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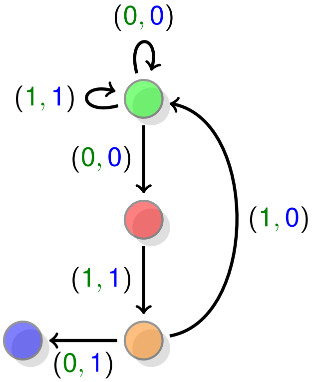

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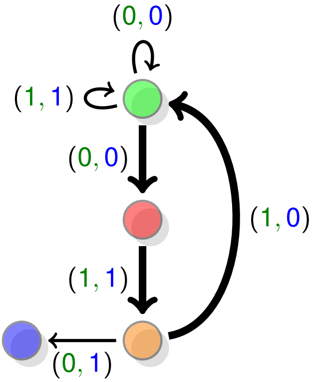

Maximum cycle ratio = $\frac{0+1+1}{0+1+0} = 2$

Tool Implementation

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- Implemented in Java, called Relacs
- Interface for replacement policies

- Fully automatic
- Provides example sequences for competitive ratio and constant

- Analysis usually practically feasible up to associativity 8
  - limited by memory consumption
  - depends on similarity of replacement policies

Online version:

`http://rw4.cs.uni-sb.de/~reineke/relacs`

Generalizations

Identified patterns and proved generalizations by hand.

Aided by example sequences generated by tool.

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$\longrightarrow$ LRU-*may*-analysis can be used for FIFO and MRU
$\longrightarrow$ optimal with respect to predictability metric Evict

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FIFO-*may*-analysis used in the analysis of the branch target buffer of the MOTOROLA POWERPC 56X.

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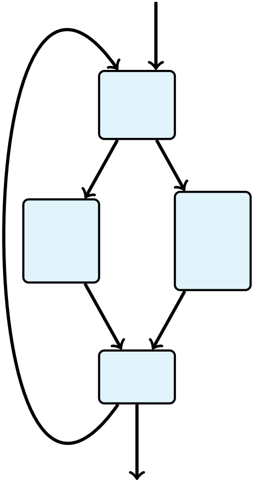
Measurement-Based Timing Analysis

- Run program on a number of inputs and initial states.
- Combine measurements for basic blocks to obtain WCET estimation.
- Sensitivity Analysis demonstrates this approach may be dramatically wrong.



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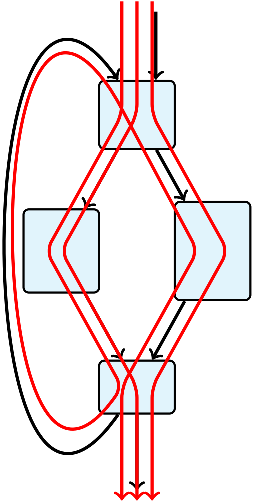
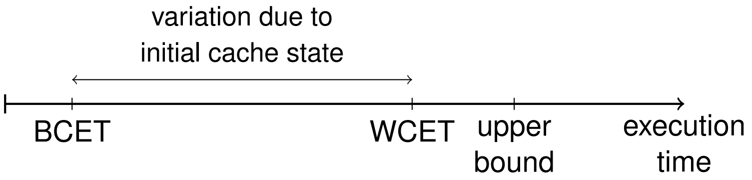


Influence of Initial Cache State



Definition (Miss sensitivity)

Policy **P** is (k, c)-miss-sensitive if

$$m_{\mathbf{P}}(p, s) \leq k \cdot m_{\mathbf{P}}(p', s) + c$$

for all access sequences $s \in M^*$ and cache-set states $p, p' \in C^{\mathbf{P}}$.

Sensitivity Results

Policy	2	3	4	5	6	7	8
LRU	$1, 2$	$1, 3$	$1, 4$	$1, 5$	$1, 6$	$1, 7$	$1, 8$
FIFO	$2, 2$	$3, 3$	$4, 4$	$5, 5$	$6, 6$	$7, 7$	$8, 8$
PLRU	$1, 2$	$-$	∞	$-$	$-$	$-$	∞
MRU	$1, 2$	$3, 4$	$5, 6$	$7, 8$	MEM	MEM	MEM

- LRU is optimal. Performance varies in the least possible way.
- For FIFO, PLRU, and MRU the number of misses may vary strongly.
- Case study based on simple model of execution time by Hennessy and Patterson (2003):
  WCET may be 3 times higher than a measured execution time for 4-way FIFO.

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Most-Recently-Used – MRU

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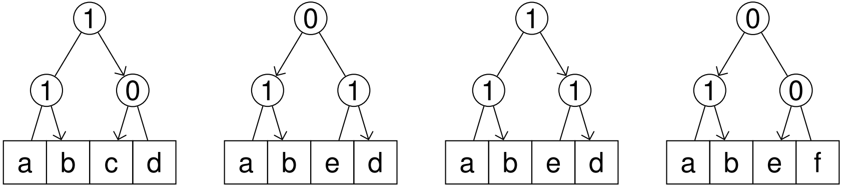
MRU-bits record whether line was recently used

$$\begin{tikzcd} \boxed{[abcd]_{0101}} \arrow[loop right, "{b,d}"] \arrow[d, bend right, "e"'] \\ \boxed{[ebcd]_{1101}} \arrow[loop right, "{e,b,d}"] \arrow[d, bend right, "c"'] \\ \boxed{[ebcd]_{0010}} \arrow[loop right, "c"] \end{tikzcd}$$

$\longrightarrow$ Never converges

Pseudo-LRU – PLRU



Initial cache-set state $[a, b, c, d]_{110}$.

After a miss on e. State: $[a, b, e, d]_{011}$.

After a hit on a. State: $[a, b, e, d]_{111}$.

After a miss on f. State: $[a, b, e, f]_{010}$.

Hit on *a* “rejuvenates” neighborhood; “saves” *b* from eviction.

May- and Must-Information

$$
\begin{aligned}
May^{\mathbf{P}}(s) &:= \bigcup_{p \in C^{\mathbf{P}}} CC_{\mathbf{P}}(update_{\mathbf{P}}(p, s)) \\
Must^{\mathbf{P}}(s) &:= \bigcap_{p \in C^{\mathbf{P}}} CC_{\mathbf{P}}(update_{\mathbf{P}}(p, s))
\end{aligned}
$$

$$
\begin{aligned}
may^{\mathbf{P}}(n) &:= \left| May^{\mathbf{P}}(s) \right|, \text{where } s \in S^{\neq} \subsetneq M^*, |s| = n \\
must^{\mathbf{P}}(n) &:= \left| Must^{\mathbf{P}}(s) \right|, \text{where } s \in S^{\neq} \subsetneq M^*, |s| = n
\end{aligned}
$$

$S^{\neq}$: set of finite access sequences with pairwise different accesses

Definitions of Metrics

$$\begin{aligned}\mathrm{Evict}^{\mathbf{P}} &:= \min\left\{n \mid \mathit{may}^{\mathbf{P}}(n) \leq n\right\},\\ \mathrm{Fill}^{\mathbf{P}} &:= \min\left\{n \mid \mathit{must}^{\mathbf{P}}(n) = k\right\},\end{aligned}$$

where k is **P**'s associativity.

Relation: Pred. Metrics $\leftrightarrow$ Rel. Competitiveness

Let $P(k)$ be $(1, 0)$-miss-competitive relative to policy $Q(l)$, then

(i) $Evict^P(k) \geq Evict^Q(l)$,

(ii) $mls^P(k) \geq mls^Q(l)$.

Alternative Pred. Metrics $\leftrightarrow$ Rel. Competitiveness

Let l be the smallest associativity, such that $\mathrm{LRU}(l)$ is $(1, 0)$-miss-competitive relative to $P(k)$. Then

$$\text{Alt-Evict}^P(k) = l.$$

Let l be the greatest associativity, such that $P(k)$ is $(1, 0)$-miss-competitive relative to $\mathrm{LRU}(l)$. Then

$$\text{Alt-mls}^P(k) = l.$$

Size of Transition System

$$\underbrace{2^{l+l'}}_{\substack{\text{status bits}\\ \text{of } \mathbf{P} \text{ and } \mathbf{Q}}} \cdot \underbrace{\sum_{i=0}^{k} \binom{k}{i}}_{\text{non-empty lines in } \mathbf{P}} \cdot \underbrace{\sum_{i'=0}^{k'} \binom{k'}{i'}}_{\text{non-empty lines in } \mathbf{Q}} \cdot \underbrace{\sum_{j=0}^{\min\{i,i'\}} \binom{i}{j}\binom{i'}{j} j!}_{\substack{\text{number of overlappings}\\ \text{in non-empty lines}}}$$

$$\begin{aligned}\sum_{j=0}^{\min\{k,k'\}} \binom{k}{j}\binom{k'}{j} j! &\leq k! \cdot k'! \sum_{j=0}^{\min\{k,k'\}} \frac{1}{(k-j)!j!(k'-j)!} \\ &\leq k! \cdot k'! \sum_{j=0}^{\infty} \frac{1}{j!} = e \cdot k! \cdot k'!\end{aligned}$$

This can be bounded by

$$2^{l+l'+k+k'} \leq |(C_k^l \times C_{k'}^{l'})/\approx| \leq 2^{l+l'+k+k'} \cdot \underbrace{e \cdot k! \cdot k'!}_{\text{bound on number of overlappings}}$$

Compatible States

$$
\begin{tikzcd}
\boxed{i^{\mathbf{P}} = [\perp\perp\perp\perp]_{\mathbf{P}}} \arrow[d, squiggly, "\mathit{update}_{\mathbf{P}}(i^{\mathbf{P}}, s)"] & \approx & \boxed{i^{\mathbf{Q}} = [\perp\perp\perp\perp]_{\mathbf{Q}}} \arrow[d, squiggly, "\mathit{update}_{\mathbf{Q}}(i^{\mathbf{Q}}, s)"] \\
\boxed{p} & \approx & \boxed{q}
\end{tikzcd}
$$

Let **P** be $(1,0)$-competitive relative to **Q**, then

$$
\begin{tikzcd}
\boxed{p} \arrow[r, phantom, "\approx"] \arrow[d, "m_{\mathbf{P}}(p, \langle x \rangle) = 1"'] \arrow[dr, phantom, "\Longrightarrow"] & \boxed{q} \arrow[d, "m_{\mathbf{Q}}(q, \langle x \rangle) = 1"] \\
\boxed{p'} \arrow[r, phantom, "\approx"] & \boxed{q'}
\end{tikzcd}
$$

$(1,0)$-Competitiveness and May/Must-Analyses

$$\begin{tikzcd}
C^{\mathbf{P}} \arrow[d, "s"'] & \approx & C^{\mathbf{Q}} \arrow[d, "s"] \\
P \arrow[d] & \approx & Q \arrow[d] \\
P' & \approx & Q'
\end{tikzcd}$$

$$\forall p \in P : m_{\mathbf{P}}(p, \langle x \rangle) = 1 \implies \forall q \in Q : m_{\mathbf{Q}}(q, \langle x \rangle) = 1$$

Case Study: Impact of Sensitivity

- Simple model of execution time from Hennessy & Patterson (2003)
- CPI_{hit} = Cycles per instruction assuming cache hits only
- $\frac{\text{Memory accesses}}{\text{Instruction}}$ including instruction and data fetches

$$\frac{T_{wc}}{T_{meas}} = \frac{\text{CPI}_{hit} + \frac{\text{Memory accesses}}{\text{Instruction}} \times \text{Miss rate}_{wc} \times \text{Miss penalty}}{\text{CPI}_{hit} + \frac{\text{Memory accesses}}{\text{Instruction}} \times \text{Miss rate}_{meas} \times \text{Miss penalty}}$$

$$= \frac{1.5 + 1.2 \times 0.20 \times 50}{1.5 + 1.2 \times 0.05 \times 50} = \frac{13.5}{4.5} \qquad = 3$$